２つの分布を加算後の分布の分散は？

$E[x] = \mu_x \quad E[y] = \mu_y$ ←分布の平均値

${\sigma_x}^2 = E[(x-\mu_x)^2] \quad {\sigma_y}^2 = E\left[\left(y-\mu_y\right)^2\right]$ ←分布の分散

$z = x + y$の分布を考えると、

$E[z] = \mu_z = \mu_x + \mu_y$

$${\sigma_z}^2 = E[(z-\mu_z)^2] = E\left[\left\{(x+y)-\left(\mu_x+\mu_y\right)\right\}^2\right] = E\left[\left\{(x-\mu_x)+\left(y-\mu_y\right)\right\}^2\right]$$

$$= E[(x-\mu_x)^2] + E\left[\left(y-\mu_y\right)^2\right] + 2E\left[(x-\mu_x)\left(y-\mu_y\right)\right]$$

$$= {\sigma_x}^2 + {\sigma_y}^2 + 2E\left[xy - \mu_x y - \mu_y x + \mu_x\mu_y\right]$$

$$= {\sigma_x}^2 + {\sigma_y}^2 + 2\left\{E[xy] - \mu_x E[y] - \mu_y E[x] + \mu_x\mu_y\right\}$$

$$= {\sigma_x}^2 + {\sigma_y}^2 + 2\left\{E[xy] - \mu_x\mu_y - \mu_y\mu_x + \mu_x\mu_y\right\}$$

←xとyが独立の場合

$E[xy] = E[x]\,E[y]$

$$= {\sigma_x}^2 + {\sigma_y}^2 + 2\,E[x]\,E[y] - 2\mu_x\mu_y$$

$$= {\sigma_x}^2 + {\sigma_y}^2 + 2\,\mu_x\mu_y - 2\mu_x\mu_y$$

$$= {\sigma_x}^2 + {\sigma_y}^2$$

2つの分布を減算後の分布の分散は？

$E[x] = \mu_x \qquad E[y] = \mu_y$ ←分布の平均値

${\sigma_x}^2 = E[(x-\mu_x)^2] \qquad {\sigma_y}^2 = E\left[\left(y-\mu_y\right)^2\right]$ ←分布の分散

$z = x - y$の分布を考えると、

$E[z] = \mu_z = \mu_x - \mu_y$

$${\sigma_z}^2 = E[(z-\mu_z)^2] = E\left[\left\{(x-y)-\left(\mu_x-\mu_y\right)\right\}^2\right] = E\left[\left\{(x-\mu_x)-\left(y-\mu_y\right)\right\}^2\right]$$

$$= E[(x-\mu_x)^2] + E\left[\left(y-\mu_y\right)^2\right] - 2E[(x-\mu_x)\left(y-\mu_y\right)]$$

$$= {\sigma_x}^2 + {\sigma_y}^2 - 2E\left[xy - \mu_x y - \mu_y x + \mu_x\mu_y\right]$$

$$= {\sigma_x}^2 + {\sigma_y}^2 - 2\{E[xy] - \mu_x E[y] - \mu_y E[x] + \mu_x\mu_y\}$$

$$= {\sigma_x}^2 + {\sigma_y}^2 - 2\{E[xy] - \mu_x\mu_y - \mu_y\mu_x + \mu_x\mu_y\}$$

←xとyが独立の場合

$E[xy] = E[x]\,E[y]$

$$= {\sigma_x}^2 + {\sigma_y}^2 - 2\,E[x]\,E[y] + 2\mu_x\mu_y$$

$$= {\sigma_x}^2 + {\sigma_y}^2 - 2\,\mu_x\mu_y + 2\mu_x\mu_y$$

$$= {\sigma_x}^2 + {\sigma_y}^2$$

分散

$$V(X) = \int_{-\infty}^{\infty}(X-\mu)^2 f(X)\,dX$$
$$= \int_{-\infty}^{\infty}(X^2 - 2X\mu + \mu^2) f(X)\,dX$$
$$= \int_{-\infty}^{\infty} X^2 f(X)\,dX - \int_{-\infty}^{\infty} 2X\mu f(X)\,dX + \int_{-\infty}^{\infty} \mu^2 f(X)\,dX \quad \leftarrow \int_{-\infty}^{\infty} f(X)\,dX = 1$$
$$= E(X^2) - 2\mu \int_{-\infty}^{\infty} X f(X)\,dX + \mu^2$$
$$= E(X^2) - 2\mu \cdot E(X) + \mu^2$$
$$= E(X^2) - 2\{E(X)\}^2 + \{E(X)\}^2 \quad \leftarrow E(X) = \mu$$
$$= E(X^2) - \{E(X)\}^2$$

$$V(X) = E(X^2) - \{E(X)\}^2$$

Xに$X+Y$を代入すると

$$V(X+Y) = E[(X+Y)^2] - \{E(X+Y)\}^2$$
$$= E[X^2 + 2XY + Y^2] - \{E(X) + E(Y)\}^2$$
$$= E(X^2) + 2E(XY) + E(Y^2) - \{E(X)\}^2 - 2E(X)E(Y) - \{E(Y)\}^2$$
$$= E(X^2) - \{E(X)\}^2 + E(Y^2) - \{E(Y)\}^2$$
$$= V(X) + V(Y)$$

Xに$X-Y$を代入すると

$$V(X-Y) = E[(X-Y)^2] - \{E(X-Y)\}^2$$
$$= E[X^2 - 2XY + Y^2] - \{E(X) - E(Y)\}^2$$
$$= E(X^2) - 2E(XY) + E(Y^2) - \{E(X)\}^2 + 2E(X)E(Y) - \{E(Y)\}^2$$
$$= E(X^2) - \{E(X)\}^2 + E(Y^2) - \{E(Y)\}^2$$
$$= V(X) + V(Y)$$

$$E[xy] = E[x]\,E[y]$$

変数X	x_1	x_2	x_3
確率P	p_1	p_2	p_3

期待値

$$E(X) = x_1p_1 + x_2p_2 + x_3p_3$$

変数X	y_1	y_2	y_3
確率Q	q_1	q_2	q_3

$$E(Y) = y_1q_1 + y_2q_2 + y_3q_3$$

	y_1	y_2	y_3	計
x_1	$\boldsymbol{p_1q_1}$	p_1q_2	p_1q_3	$\boldsymbol{p_1}$
x_2	$\boldsymbol{p_2q_1}$	p_2q_2	p_2q_3	$\boldsymbol{p_2}$
x_3	$\boldsymbol{p_3q_1}$	p_3q_2	p_3q_3	$\boldsymbol{p_3}$
計	$\boldsymbol{q_1}$	q_2	q_3	**1**

$$p_1q_1 + p_2q_1 + p_3q_1 = (p_1 + p_2 + p_3)\,q_1 = \boldsymbol{q_1}$$

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$$\begin{aligned}
E(XY) &= x_1y_1p_1q_1 + x_1y_2p_1q_2 + x_1y_3p_1q_3 \\
&\quad + x_2y_1p_2q_1 + x_2y_2p_2q_2 + x_2y_3p_2q_3 \\
&\quad + x_3y_1p_3q_1 + x_3y_2p_3q_2 + x_3y_3p_3q_3 \\
&= x_1\,p_1(y_1q_1 + y_2q_2 + y_3q_3) + \\
&\quad x_2\,p_2(y_1q_1 + y_2q_2 + y_3q_3) + \\
&\quad x_3\,p_3(y_1q_1 + y_2q_2 + y_3q_3) + \\
&= (x_1\,p_1 + x_2\,p_2 + x_3\,p_3)\,(y_1q_1 + y_2q_2 + y_3q_3) \\
&= E(X)\,E(Y)
\end{aligned}$$