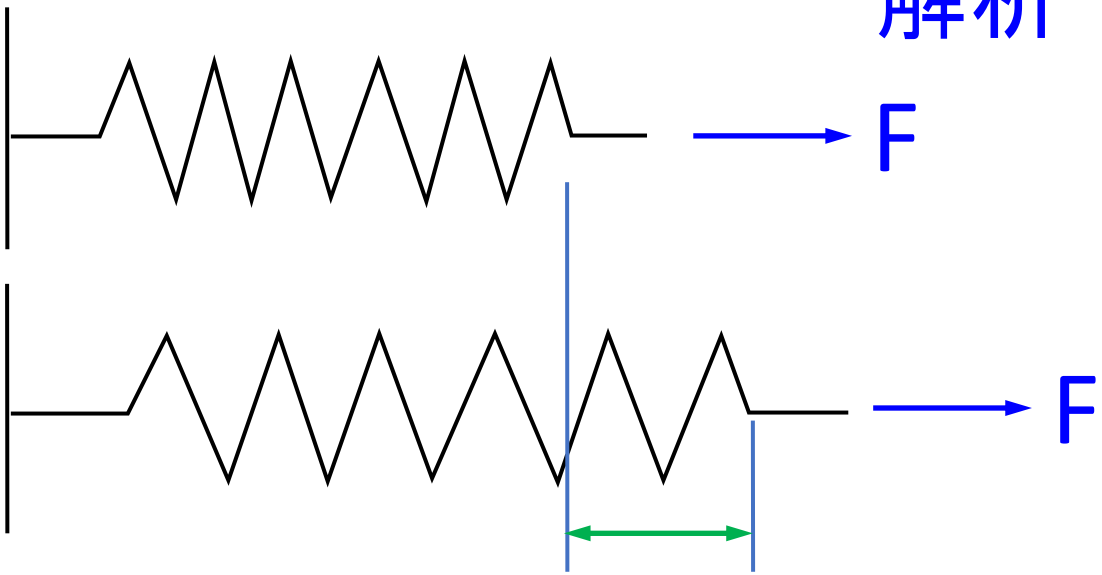
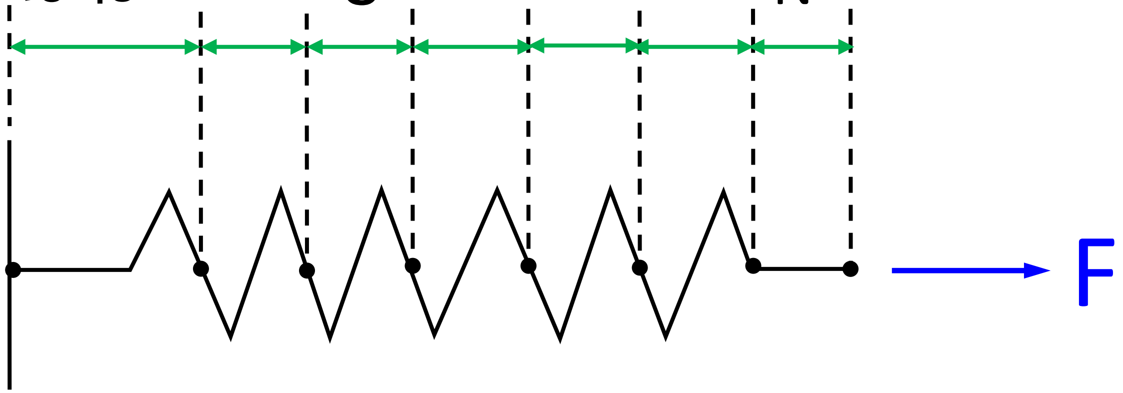
解析



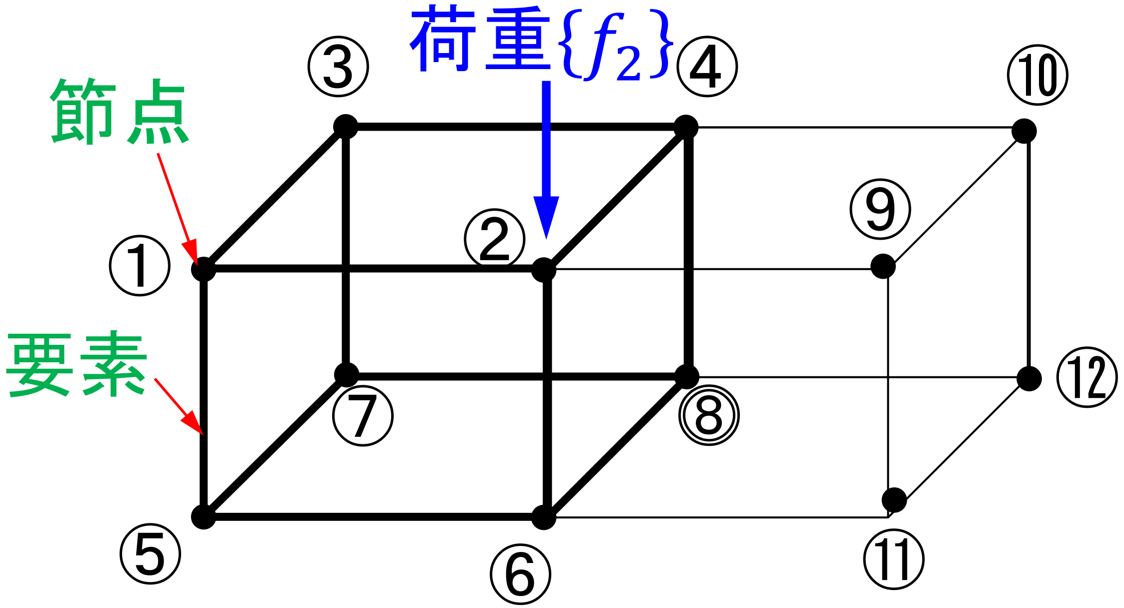
$$F = kd$$



$$F_1 = k_1 d_1$$
$$F_2 = k_2 d_2$$
$$F_3 = k_3 d_3$$
$$\cdots\cdots\cdots\cdots$$
$$F_N = k_N d_N$$

$$\{F\} = [K]\{u\}$$

$\{F\}$: 荷重ベクトル
$[K]$: 全体剛性マトリクス
$\{u\}$: 変位ベクトル



$$\{u\} = \begin{Bmatrix} u_1 \\ \vdots \\ u_n \end{Bmatrix} \qquad \{f\} = \begin{Bmatrix} f_1 \\ \vdots \\ f_n \end{Bmatrix}$$

$$[K] = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix}$$

$\{F\} = [K]\{u\}$より

変位ベクトル$\{u\} = [K]^{-1}\{F\}$

$\sigma = E\varepsilon$ ⇨ $\{\sigma\} = [D]\{\varepsilon\}$

σ:応力

E:弾性係数

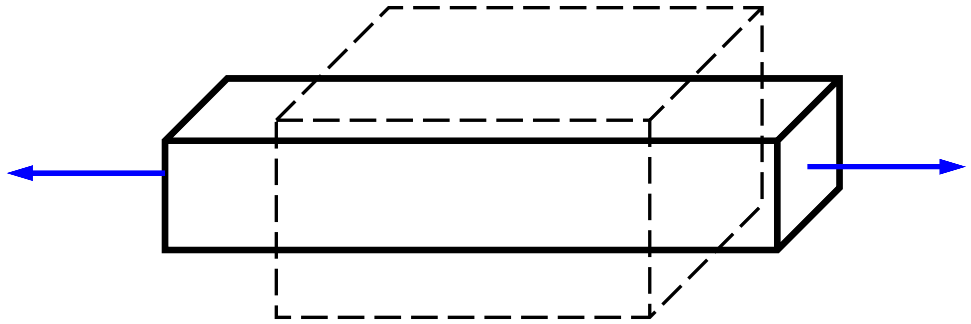
ε:ひずみ

Dマトリクス:材料の硬さを表す行列

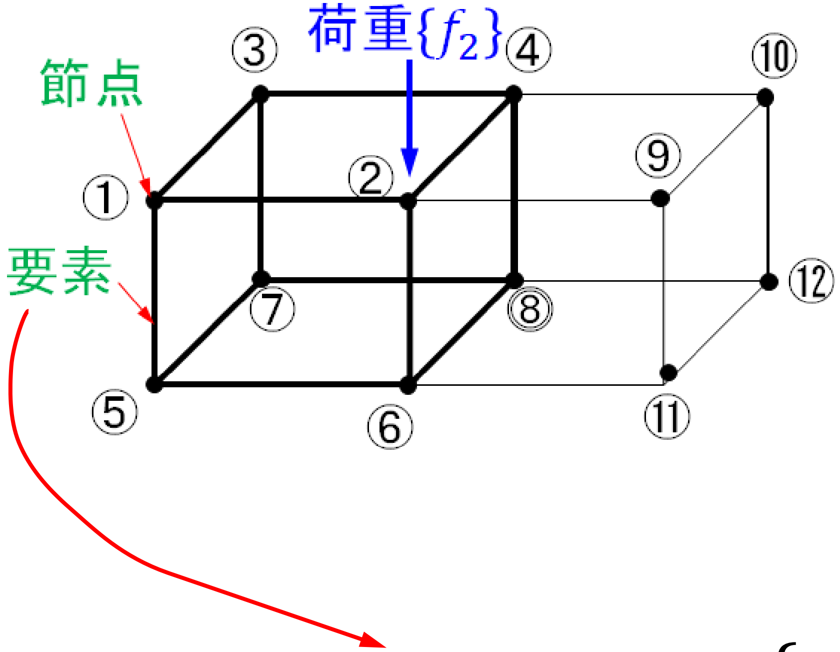
1次元

$$\sigma = E\varepsilon$$

3次元



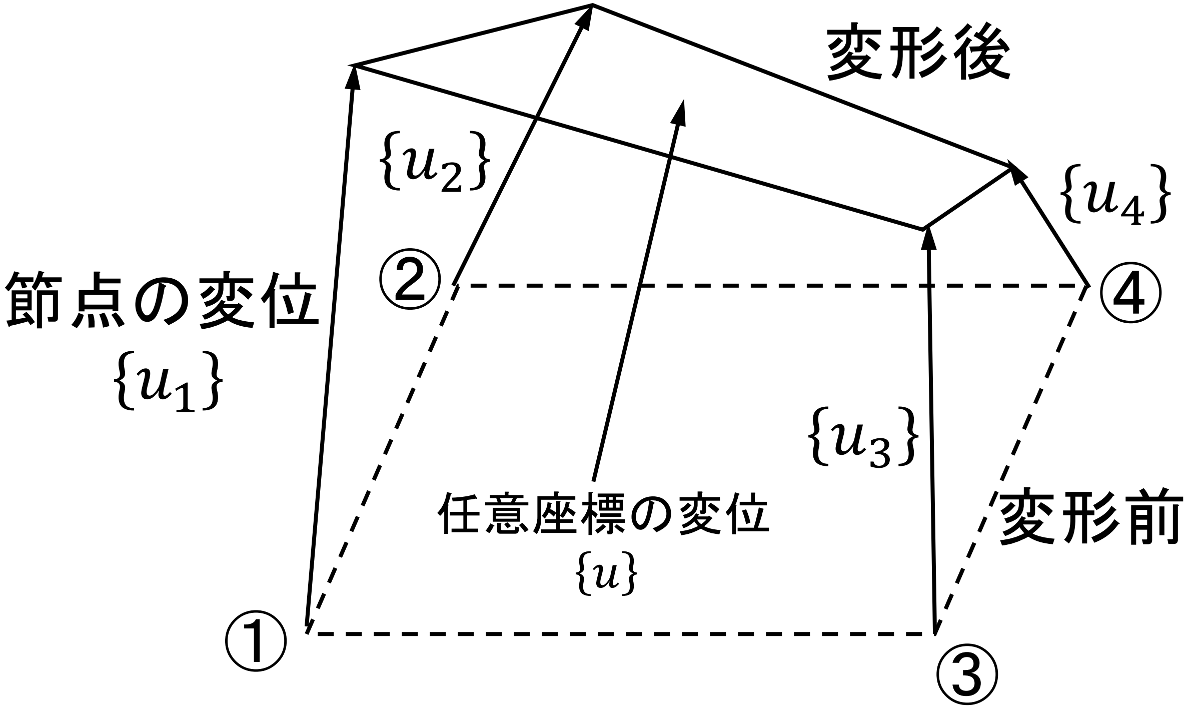
$$\{\sigma\} = [D]\{\varepsilon\}$$



N: 形状関数　要素内の物理量分布

1要素当たり8個の節点の変位 $\{u\} = [N_1]\{u_1\} + [N_2]\{u_2\} + \cdots + [N_8]\{u_8\}$

$$= \sum_{i=1}^{8} [N_i]\{u_i\}$$

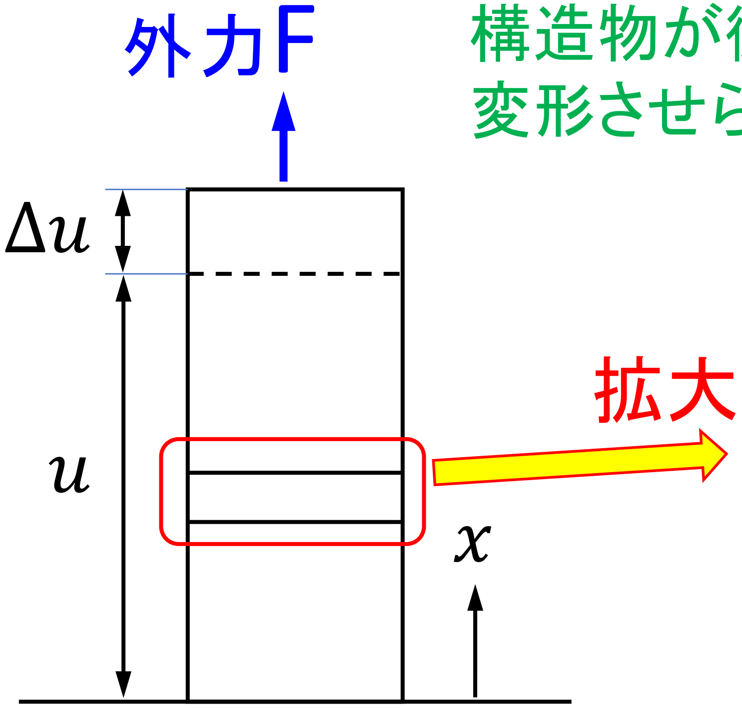


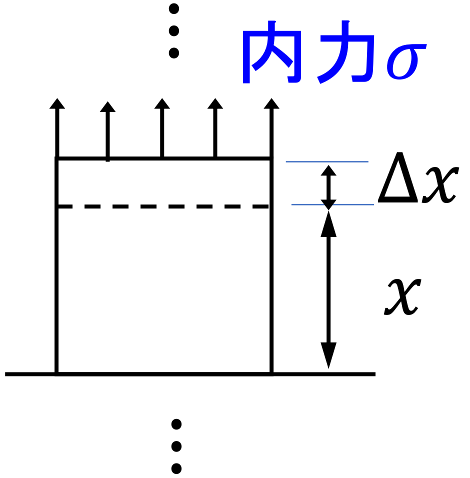
任意座標でのひずみ

$$\{\varepsilon\} = \begin{Bmatrix} \dfrac{du_x}{dx} \\ \dfrac{du_y}{dy} \\ \dfrac{du_z}{dz} \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} \dfrac{d[N_i]}{dx} \\ \dfrac{d[N_i]}{dy} \\ \dfrac{d[N_i]}{dz} \end{bmatrix} \{u_i\} = [B] \sum_{i=1}^{8} \{u_i\}$$

Bマトリクス：形状関数の微分成分の行列

構造物が微小変形したとき、外力による仕事Wは、構造物内部で変形させられるエネルギーUに等しい





$$W = \frac{1}{2} F \Delta u$$

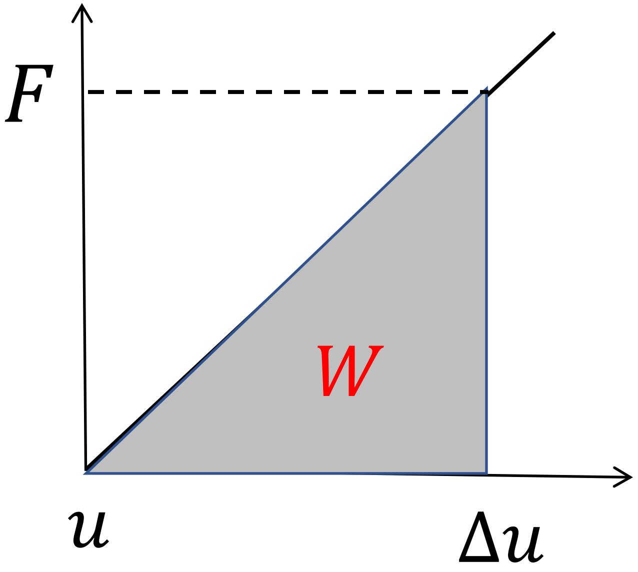
$$\sigma = \boxed{E\varepsilon = E\frac{\Delta x}{x}} = \frac{F}{A}$$ ←断面積

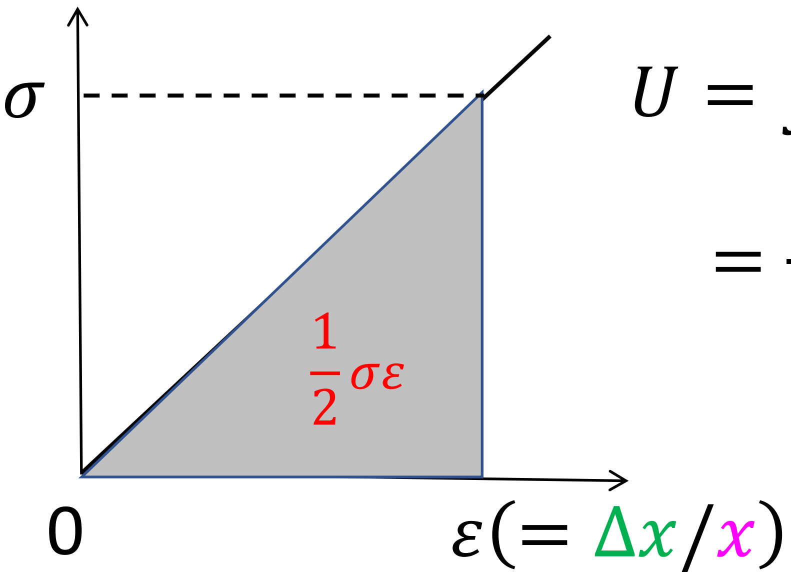
$$W = \frac{1}{2} F \varepsilon u = \frac{F^2 u}{2EA}$$

$$dU = \frac{1}{2}\sigma\varepsilon A dx$$

$$U = \int_{x=0}^{u} \frac{1}{2}\sigma\varepsilon A dx = \frac{1}{2}\int_{x=0}^{u} \frac{F}{A}\frac{F}{EA} A dx$$

$$= \frac{F^2}{2EA}\int_{x=0}^{u} dx = \frac{F^2 u}{2EA}$$





$$W = U$$

$$\boxed{\{\varepsilon\} = [B]\sum_{i=1}^{8}\{u_i\}} \qquad \{\sigma\} = [D]\{\varepsilon\}$$

$$\boxed{\{\sigma\}} = [D]\{\varepsilon\} = \boxed{[D]\,[B]\,\textstyle\sum_{i=1}^{8}\{u_i\}}$$

$$dU = \{\delta\varepsilon\}\{\sigma\}$$

$$U = \int dU\,dv$$

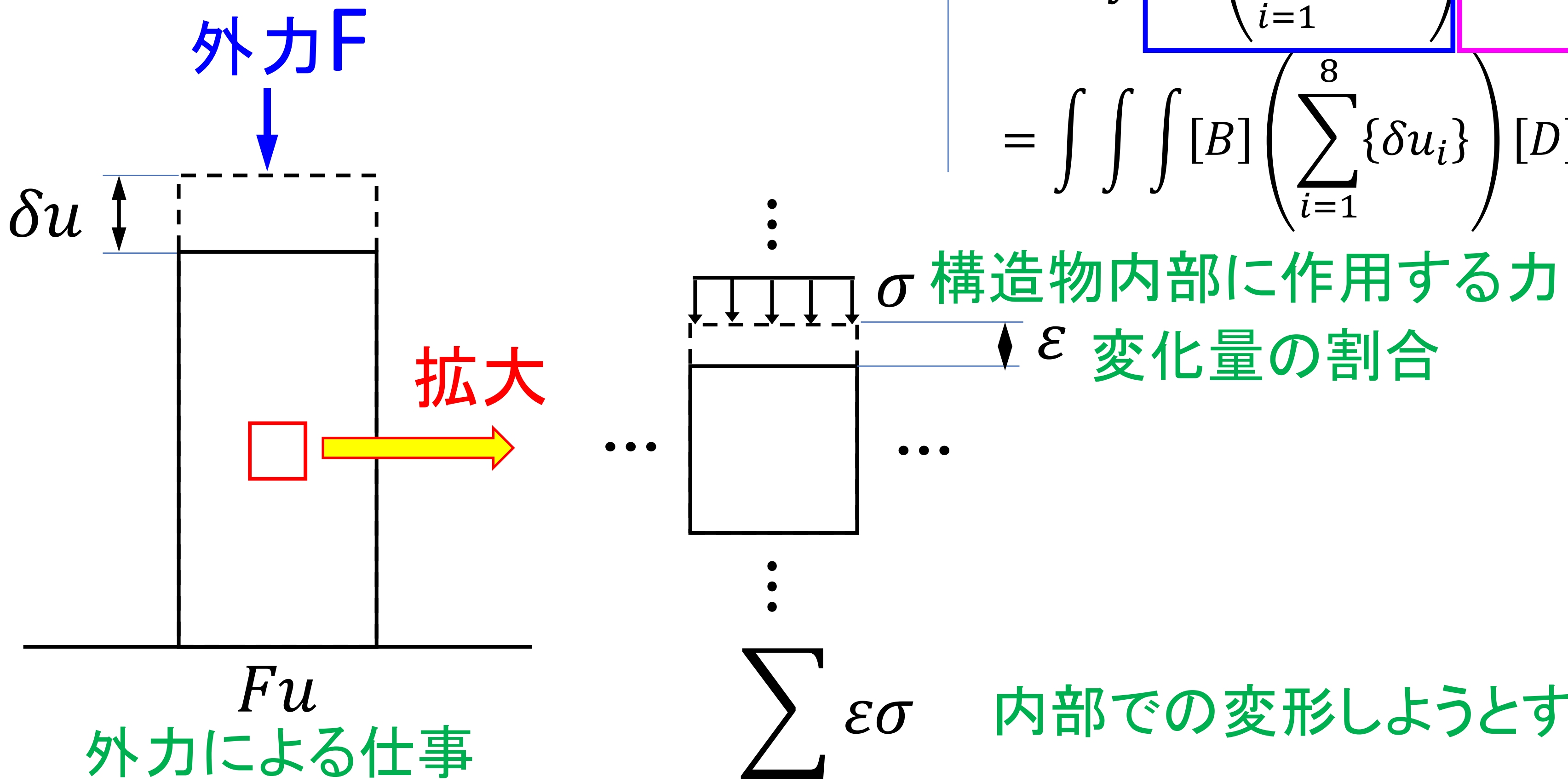
$$= \int \{\delta\varepsilon\}\{\sigma\}\,dv$$

$$= \int \boxed{[B]\left(\sum_{i=1}^{8}\{\delta u_i\}\right)}\,\boxed{[D][B]\left(\sum_{i=1}^{8}\{u_i\}\right)}dv$$

$$= \int\int\int [B]\left(\sum_{i=1}^{8}\{\delta u_i\}\right)[D][B]\left(\sum_{i=1}^{8}\{u_i\}\right)\mathrm{dxdydz}$$

↑

3次元に拡張



内部での変形しようとするエネルギー

仕事W＝（移動距離）×力なので

$W = \left(\sum_{i=1}^{8}\{\delta u_i\}\right)^T \{F\}$　←Wはスカラー量なので、$\sum_{i=1}^{8}\{\delta u_i\}$の転置行列にして$\{F\}$との内積にしている

$W = U$なので

$$\left(\sum_{i=1}^{8}\{\delta u_i\}\right)^T \{F\} = \iiint [B]\left(\sum_{i=1}^{8}\{\delta u_i\}\right)[D][B]\left(\sum_{i=1}^{8}\{u_i\}\right)dxdydz$$

ここで、$\{\delta u_i\}$、$\{u_i\}$はx、y、及びzに依存しない定数。$[B]\,[A] = [A]^T[B]^T$なので

$$\cancel{\left(\sum_{i=1}^{8}\{\delta u_i\}\right)^T} \{F\} = \cancel{\left(\sum_{i=1}^{8}\{\delta u_i\}\right)^T} \iiint [B]^T\,[D][B]dxdydz\left(\sum_{i=1}^{8}\{u_i\}\right)$$

$$\{F\} = \boxed{\iiint [B]^T\,[D][B]dxdydz}\left(\sum_{i=1}^{8}\{u_i\}\right) = \boxed{[K_e]}\left(\sum_{i=1}^{8}\{u_i\}\right) = [K_e]\{u\}$$

↑

要素剛性マトリクス

$$\{u\} = [K_e]^{-1}\{F\}$$