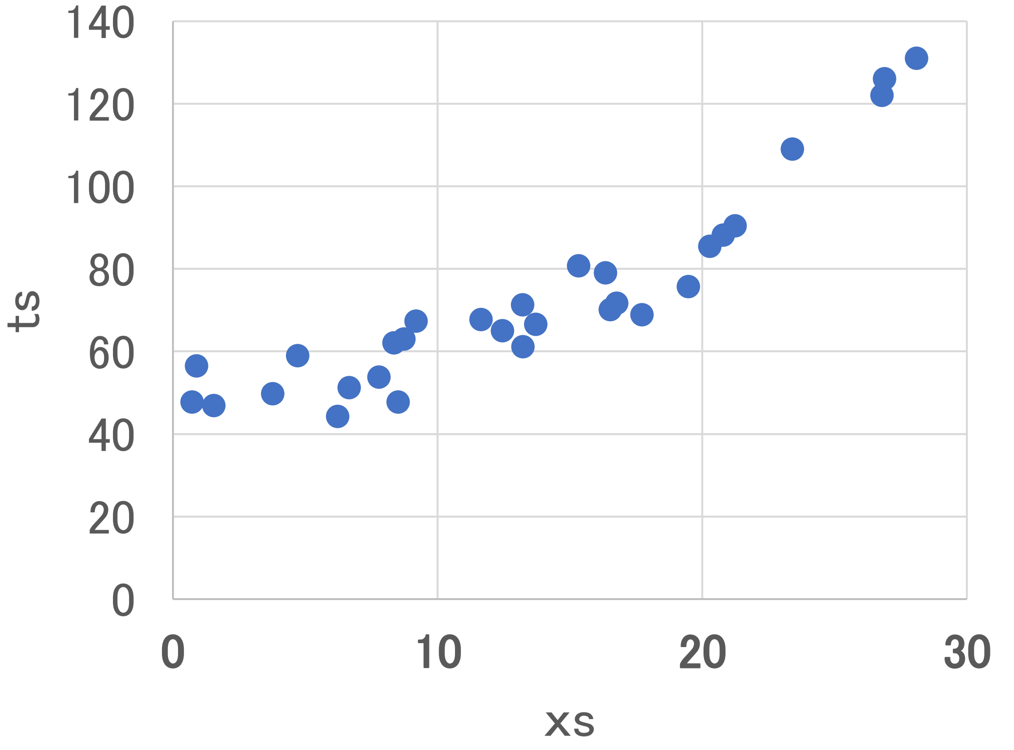
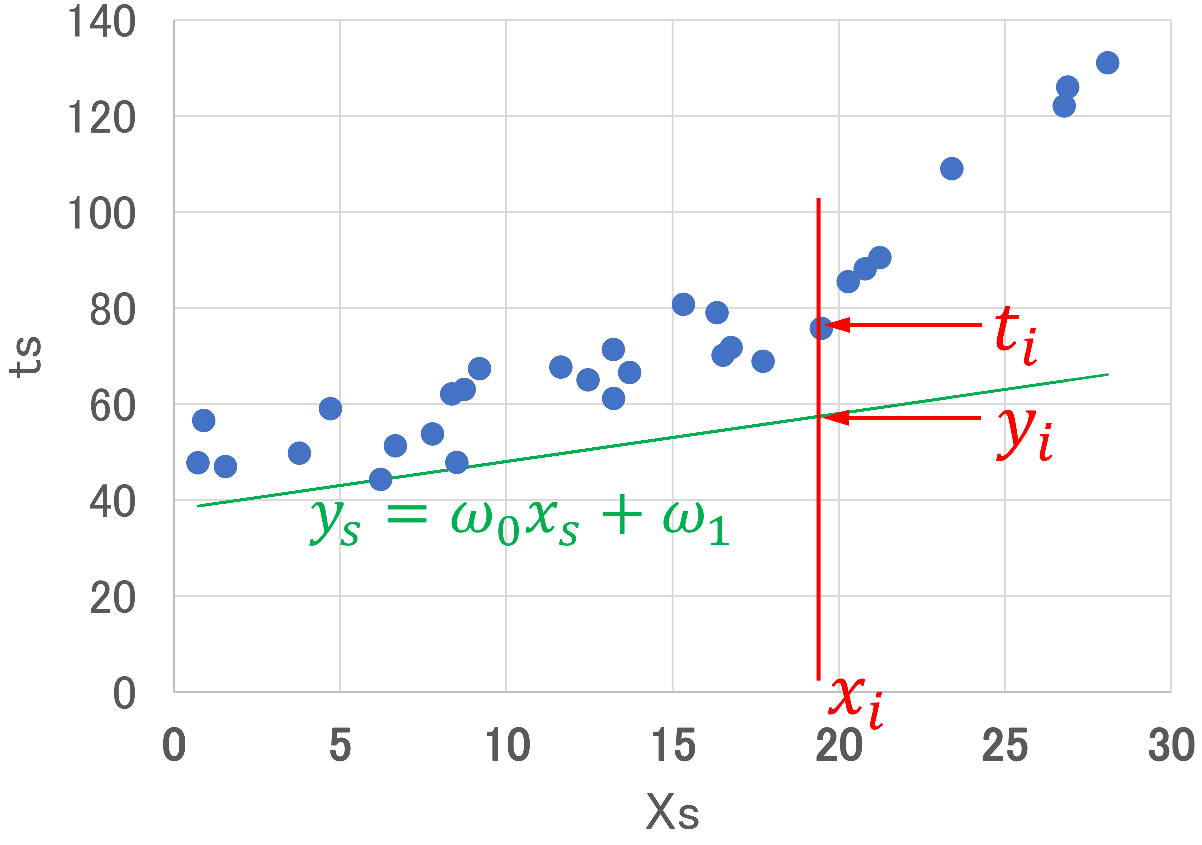
線形回帰

データ

```
xs [16.524 21.244  8.727 15.325 26.788 26.889  3.768  6.217  1.544 13.224
  0.896 13.705 19.474  8.355 20.288 17.726  0.719 16.766  7.778 12.453
  8.506 20.794 13.214  4.706 16.339 23.409  9.191  6.659 11.639 28.092]

ts [ 70.135  90.464  63.033  80.721 122.065 126.031  49.769  44.271  46.926
  61.163  56.516  66.573  75.741  62.075  85.447  68.883  47.736  71.724
  53.774  65.007  47.788  88.152  71.328  59.019  79.008 109.037  67.342
  51.264  67.704 131.06 ]
```





$$J = \frac{1}{n}\{(t_0 - y_0)^2 + (t_1 - y_1)^2 + \cdots\}$$
$$= \frac{1}{n}\sum_{i=0}^{n-1}(t_i - y_i)^2$$

平均二乗誤差（MSE：mean squared error）

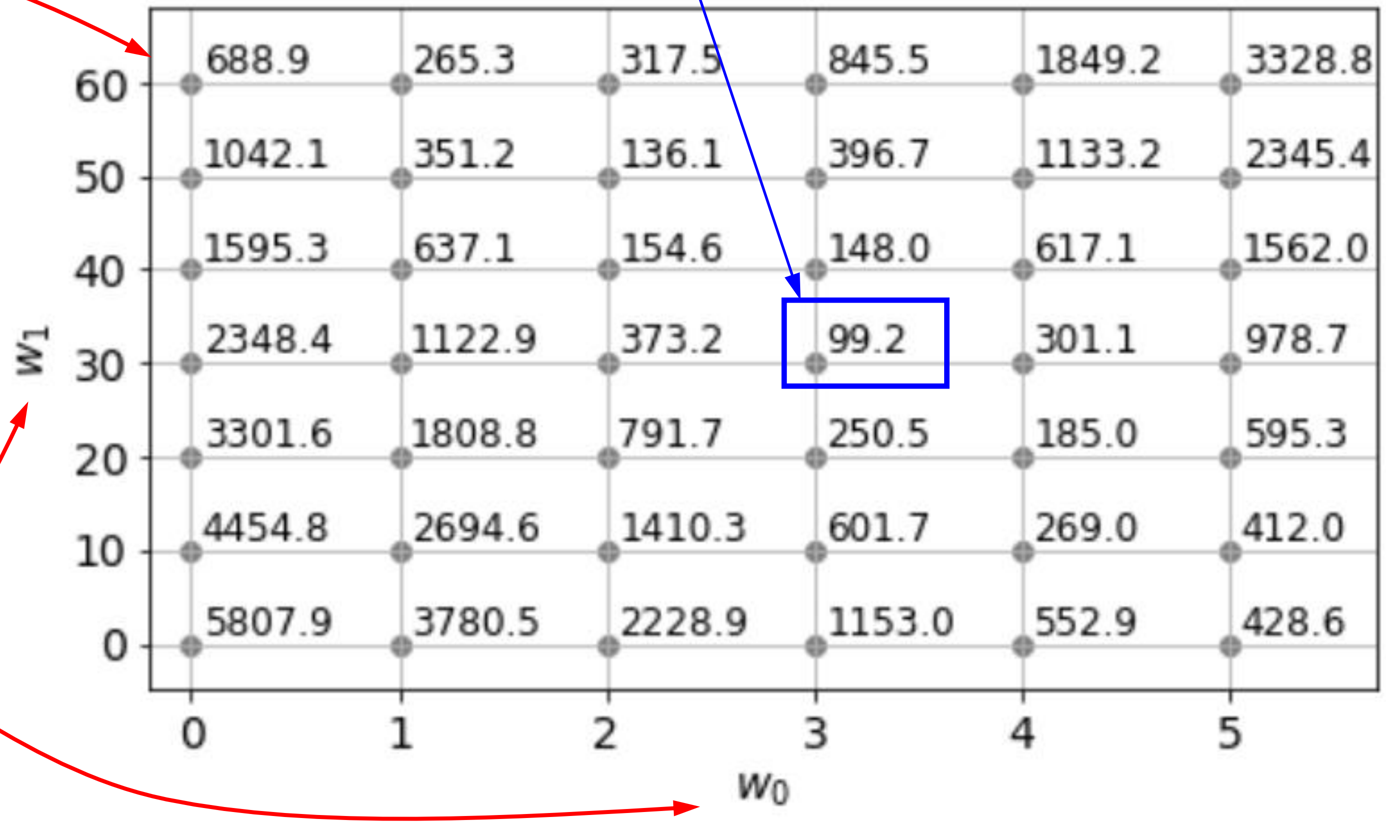
平均二乗誤差(MSE)のマトリクス

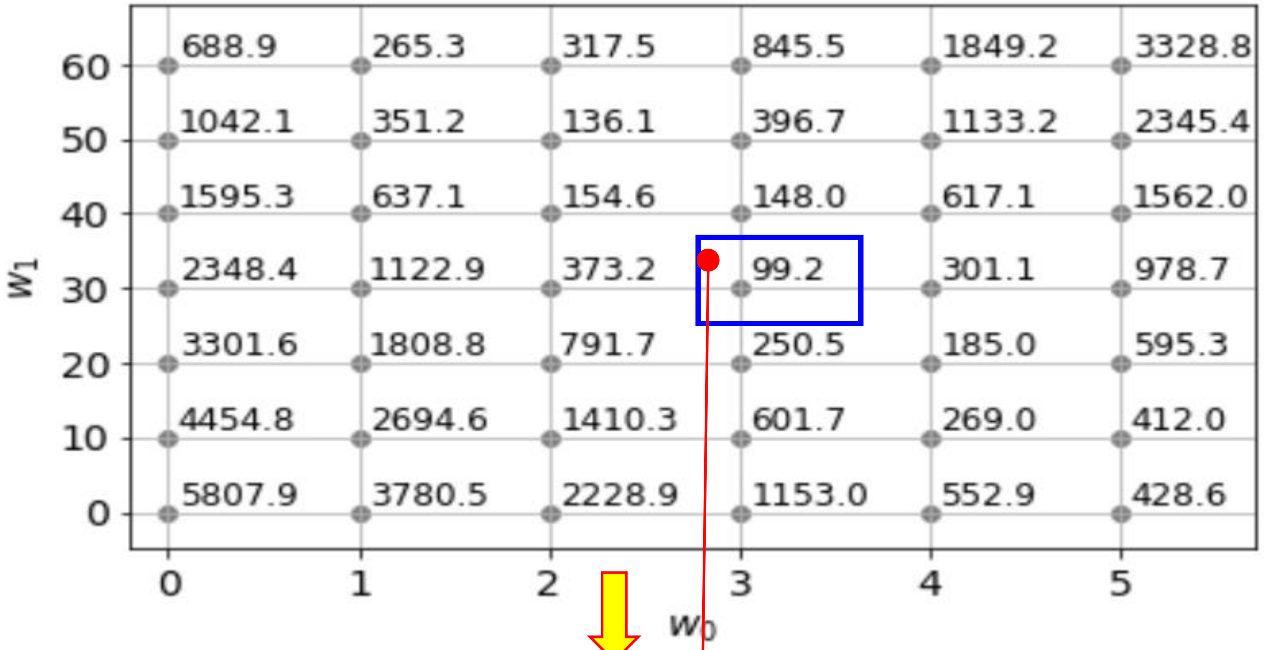
MSEが極小になるのは？

$$x_s = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} \quad t_s = \begin{bmatrix} t_0 \\ t_1 \\ \vdots \\ t_{n-1} \end{bmatrix}$$

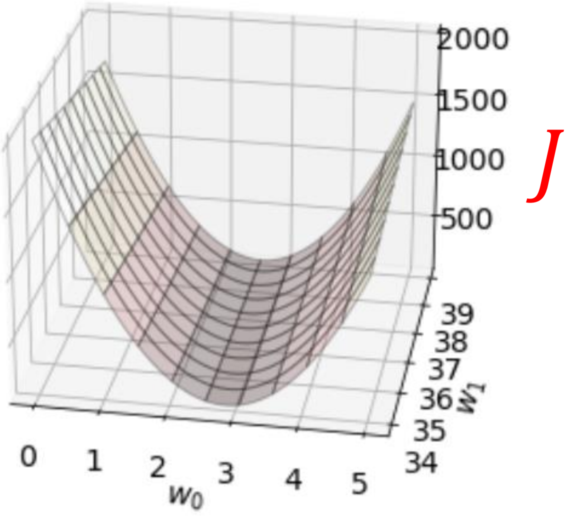
$$y_s = \omega_0 x_s + \omega_1$$

$$J = \frac{1}{n}\{(t_0 - y_0)^2 + (t_1 - y_1)^2 + \cdots\}$$
$$= \frac{1}{n}\sum_{i=0}^{n-1}(t_i - y_i)^2$$

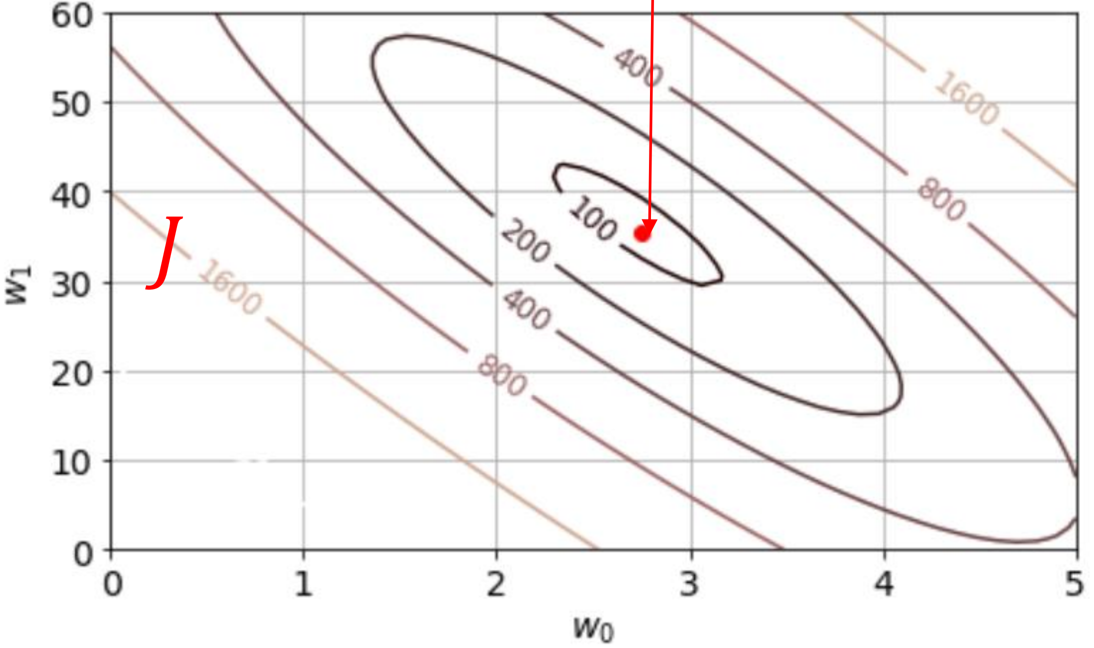


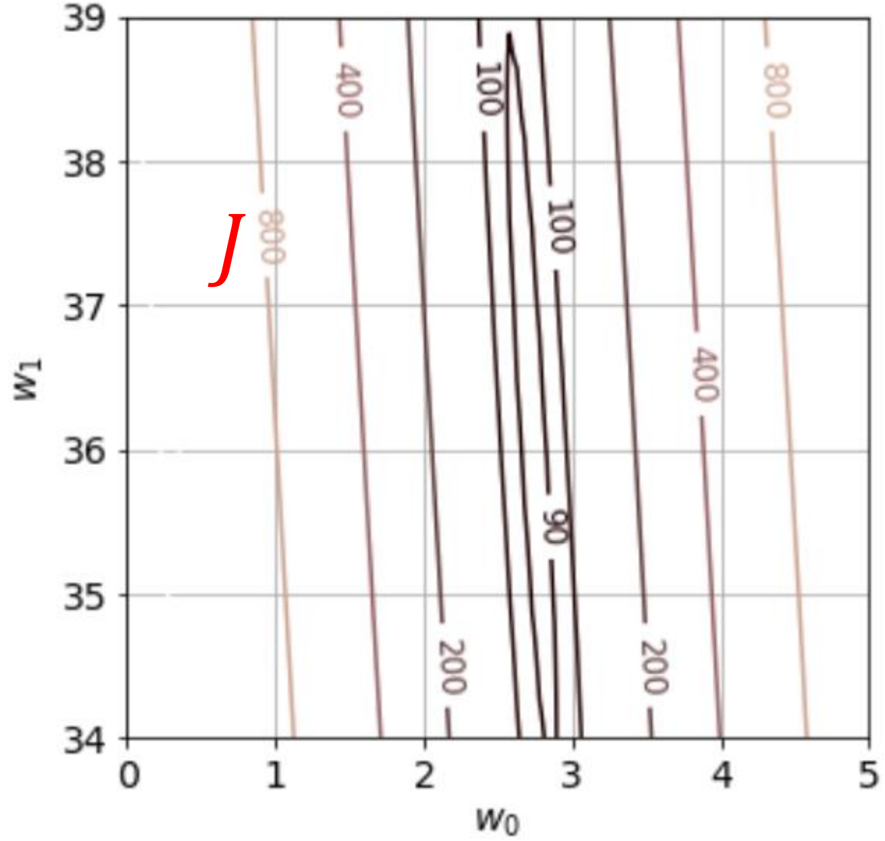


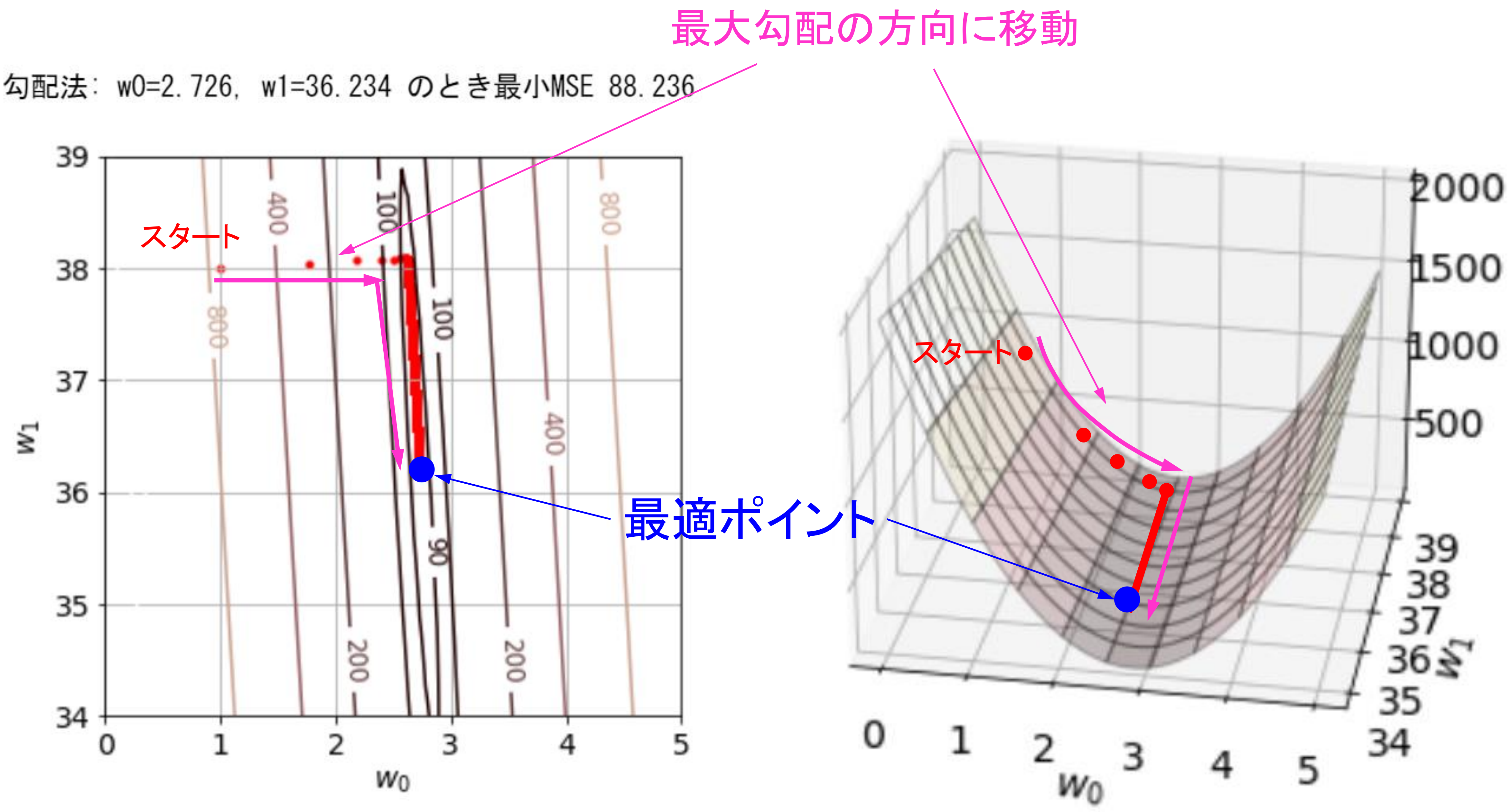
3次元で描くと



グリッドサーチ: w0=2.755, w1=35.510 のとき最小MSE 88.391





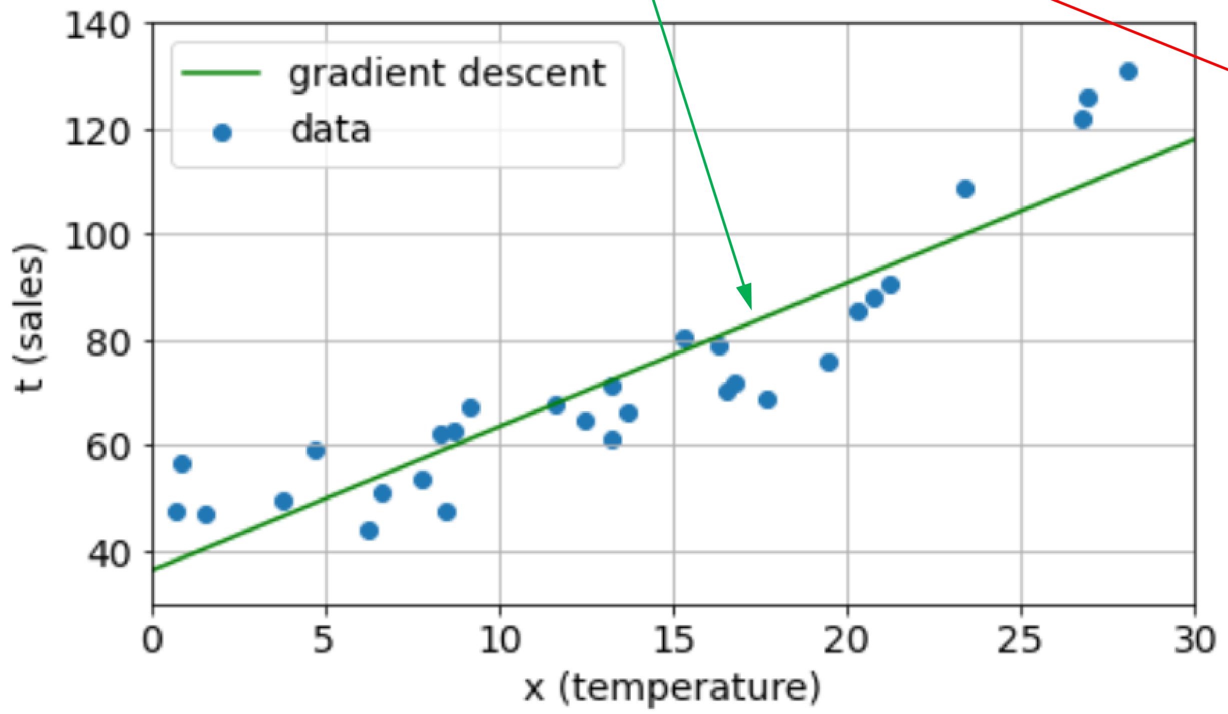


最大勾配の方向に移動
勾配法: w0=2.726, w1=36.234 のとき最小MSE 88.236
スタート
最適ポイント
w_0
w_1

最適ポイントのω_0とω_1を代入して線形回帰式

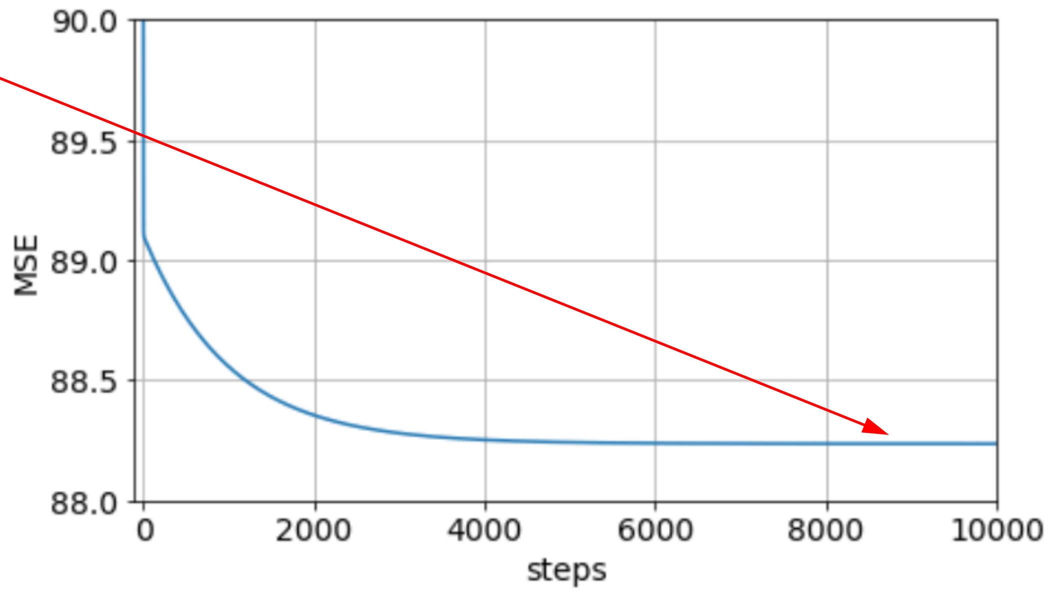
$$y = \omega_0 x + \omega_1$$

勾配法: w0=2.726, w1=36.234 のとき最小MSE 88.236



学習曲線

最小MSE = 88.24



$$x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} \quad t = \begin{bmatrix} t_0 \\ t_1 \\ \vdots \\ t_{n-1} \end{bmatrix}$$

解析解： w0=2.726, w1=36.221 のとき最小MSE 88.236

$$\begin{bmatrix} t_0 \\ t_1 \\ \vdots \\ t_{n-1} \end{bmatrix} = \begin{bmatrix} x_0 & 1 \\ x_1 & 1 \\ \vdots & \vdots \\ x_{n-1} & 1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

代数法： 996μs

勾配法： 714ms

$$t = x\,w$$

$$x^T t = x^T x\,w$$

$$(x^T x)^{-1}\,x^T t = (x^T x)^{-1} x^T x\,w$$

$$= w$$

$$w = (x^T x)^{-1}\,x^T \mathrm{t}$$

ムーア・ペンローズの疑似逆行列