オイラー定数γ

$$\gamma = \lim_{n\to\infty}\left\{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-logn\right\}$$

数学では
$logn = log_e n = \ln n$

$logn = log\left(\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\cdots\cdot\frac{n}{n-1}\right) = \sum_{k=1}^{n-1} log\left(1+\frac{1}{k}\right)$を代入して

$$\gamma = \lim_{n\to\infty}\left\{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-logn\right\}$$
$$= \sum_{k=1}^{n-1}\left\{\frac{1}{k}-log\left(1+\frac{1}{k}\right)\right\}+\frac{1}{n}$$
$$= \sum_{k=1}^{n-1}\left[\frac{1}{k}-\{log(k+1)-logk\}\right]+\frac{1}{n}$$
$$= \sum_{k=1}^{n-1}\left(\frac{1}{k}-\int_k^{k+1}\frac{dx}{x}\right)+\frac{1}{n}$$

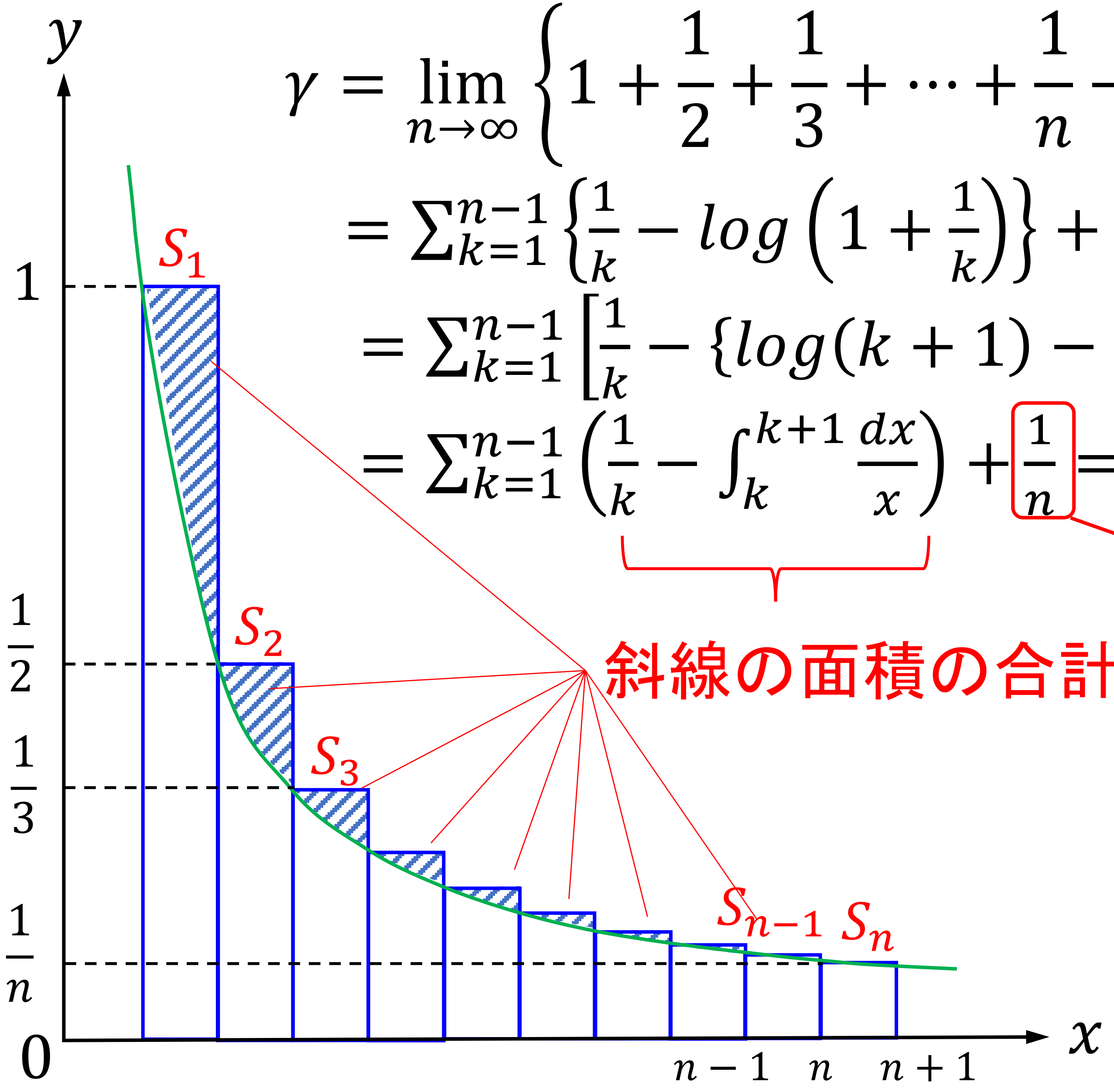
オイラー定数γ

$$\gamma = \lim_{n\to\infty}\left\{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n\right\}$$

$$= \sum_{k=1}^{n-1}\left\{\frac{1}{k} - \log\left(1 + \frac{1}{k}\right)\right\} + \frac{1}{n}$$

$$= \sum_{k=1}^{n-1}\left[\frac{1}{k} - \{\log(k+1) - \log k\}\right] + \frac{1}{n}$$

$$= \sum_{k=1}^{n-1}\left(\frac{1}{k} - \int_k^{k+1}\frac{dx}{x}\right) + \frac{1}{n} = 0.5772$$



$\ln n$の場合は$\gamma = 0.5772$
$0.5772/ln10 = 0.2507$

$n \to \infty$のとき$\frac{1}{n} \to 0$

短冊面積と$log n$の間には
0.5772の誤差がある