連立方程式

連立方程式を解く

$$x_1 + 2x_2 + 3x_3 = 8$$
$$2x_1 + 5x_2 + 7x_3 = 5$$
$$3x_1 + 7x_2 + 11x_3 = 9$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 11 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 8 \\ 5 \\ 9 \end{Bmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 11 \end{pmatrix} \quad x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \quad b = \begin{Bmatrix} 8 \\ 5 \\ 9 \end{Bmatrix}$$

$Ax = \mathrm{b}$　　$A^{-1}Ax = A^{-1}\,\mathrm{b}$　　$x = A^{-1}\,\mathrm{b}$

	A	B	C	D	E
	A				b
	1	2	3		8
	2	5	7		5
	3	7	11		9
	A^{-1}				
	=MINVERSE(A2:C4)				

①枠指定　②　＝MINVERSE(A1:C4)　を入力　③　Shift＋Ctrl＋Enter

実行結果

A^{-1}		
6	−1	−1
−1	2	−1
−1	−1	1

A				b
1	2	3		8
2	5	7		5
3	7	11		9
A^{-1}				x
6	−1	−1		=MMULT(A7:C9,E2:E4)
−1	2	−1		MMULT(配列1，配列2)
−1	−1	1		

④枠指定　⑤　＝MMULT(A7:C9, E2:E4)　を入力　⑥　Shift＋Ctrl＋Enter

実行結果

A				b
1	2	3		8
2	5	7		5
3	7	11		9
A^{-1}				x
6	−1	−1		34
−1	2	−1		−7
−1	−1	1		−4

解

$x_1 + 2x_2 + 3x_3 = 8$ …①

$2x_1 + 5x_2 + 7x_3 = 5$ …②

$3x_1 + 7x_2 + 11x_3 = 9$ …③

②−①×2、③−①×3より

$x_1 + 2x_2 + 3x_3 = 8$ …①

$x_2 + x_3 = -11$ …②′

$x_2 + 2x_3 = -15$ …③′

③′ −②′より

$x_1 + 2x_2 + 3x_3 = 8$ …①

$x_2 + x_3 = -11$ …②′

$x_3 = -4$ … ③″

③″を②′に代入して

$x_2 = -7$ … ②″

さらに、③″と②″を①に代入して

$x_1 = 34$

ガウスの消去法 前進消去

$$\begin{pmatrix} 1 & 2 & 3 & 8 \\ 2 & 5 & 7 & 5 \\ 3 & 7 & 11 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 8 \\ 0 & 1 & 1 & -11 \\ 0 & 1 & 2 & -15 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 8 \\ 0 & 1 & 1 & -11 \\ 0 & 0 & 1 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 8 \\ -11 \\ -4 \end{Bmatrix}$$

$x_1 + 2x_2 + 3x_3 = 8$

$x_2 + x_3 = -11$

$x_3 = -4$

後退代入

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = a_{1n+1}$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = a_{2n+1}$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = a_{nn+1}$$

ガウスの消去法 前進消去

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & a_{1n+1} \\ a_{21} & a_{22} & & a_{2n} & a_{2n+1} \\ \vdots & & \ddots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{nn+1} \end{pmatrix} = \begin{pmatrix} 1 & a_{12} / a_{11} & \cdots & a_{1n} / a_{11} & a_{1n+1} / a_{11} \\ a_{21} & a_{22} & & a_{2n} & a_{2n+1} \\ \vdots & & \ddots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{nn+1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & a_{12} / a_{11} & & a_{1n} / a_{11} & a_{1n+1} / a_{11} \\ 0 & a_{22} - \frac{a_{21} \cdot a_{12}}{a_{11}} & \cdots & a_{2n} - \frac{a_{21} \cdot a_{1n}}{a_{11}} & a_{2n+1} - \frac{a_{21} \cdot a_{1n+1}}{a_{11}} \\ & \vdots & \ddots & & \vdots \\ 0 & a_{n2} - \frac{a_{n1} \cdot a_{12}}{a_{11}} & \cdots & a_{nn} - \frac{a_{n1} \cdot a_{1n}}{a_{11}} & a_{nn+1} - \frac{a_{n1} \cdot a_{1n+1}}{a_{11}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & b_{12} & \cdots & b_{1n} & b_{1n+1} \\ 0 & 1 & & b_{2n} & b_{2n+1} \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & b_{nn+1} \end{pmatrix}$$

一般化

$$x_n = b_{n,n+1}$$
$$x_{n-1} = b_{n-1,n+1} - b_{n-1,n}\, x_n$$
$$x_k = b_{kn-1} - \sum_{j=k+1}^{n} b_{kj} x_j$$

後退代入

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & a_{1n+1} \\ a_{21} & a_{22} & & a_{2n} & a_{2n+1} \\ \vdots & & \ddots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{nn+1} \end{pmatrix} = \begin{pmatrix} 1 & a_{12}\,/\,a_{11} & \cdots & a_{1n}\,/\,a_{11} & a_{1n+1}\,/\,a_{11} \\ a_{21} & a_{22} & & a_{2n} & a_{2n+1} \\ \vdots & & \ddots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{nn+1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & a_{12}\,/\,a_{11} & & a_{1n}\,/\,a_{11} & a_{1n+1}\,/\,a_{11} \\ 0 & a_{22} - \frac{a_{21} \cdot a_{12}}{a_{11}} & \cdots & a_{2n} - \frac{a_{21} \cdot a_{1n}}{a_{11}} & a_{2n+1} - \frac{a_{21} \cdot a_{1n+1}}{a_{11}} \\ & \vdots & \ddots & & \vdots \\ 0 & a_{n2} - \frac{a_{n1} \cdot a_{12}}{a_{11}} & \cdots & a_{nn} - \frac{a_{n1} \cdot a_{1n}}{a_{11}} & a_{nn+1} - \frac{a_{n1} \cdot a_{1n+1}}{a_{11}} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & b_{12} & \cdots & b_{1n} & b_{1n+1} \\ 0 & 1 & & b_{2n} & b_{2n+1} \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & b_{nn+1} \end{pmatrix}$$

プログラム化準備

行列の要素を$a(i,j)$とする

k行は、

ガウスの前進消去

$$a(k,j) = \frac{a(k,j)}{a(k,k)} \quad (j = 1,2,\cdots,n+1)$$

$$a(i,j) = a(i,j) - a(i,k)a(k,j) \quad (i = k+1,\cdots,n \quad j = k+1,\cdots,n+1)$$

最後に

$$a(n,n+1) = \frac{a(n,n+1)}{a(n,n)}$$ で終了

後退代入

$$x_n = b_{n,n+1}$$

$$x_{n-1} = b_{n-1,n+1} - b_{n-1,n}\, x_n$$

$$x_k = b_{kn-1} - \sum_{j=k+1}^{n} b_{kj} x_j$$

$$x(n) = a(n,n+1)$$

$$x(k) = a(k,n+1)$$

$$x(k) = x(k) - a(k,j)x(j) \qquad (i = k+1 \sim n)$$

マクロで解く連立方程式

	A	B	C	D	E	F	G
1	n	3					
2	A				b		x
3	1	2	3		8		34
4	2	5	7		5		−7
5	3	7	11		9		−4
6							

```
Sub matrix()
Dim a(10, 11), x(10)                  ‘配列用意

n = [B1]                              ‘3を読み込む

For i = 1 To n                        ‘係数行列A作成
  For j = 1 To n
    a(i, j) = Cells(i + 2, j)         ‘係数読み込み
Next j
  a(i, n + 1) = Cells(i + 2, n + 2)   ‘定数項b読み込み
Next i

For k = 1 To n − 1                    ‘ガウス消去
  For j = k + 1 To n + 1
    a(k, j) = a(k, j) / a(k, k)
  Next j
  For i = k + 1 To n
    For j = k + 1 To n + 1
      a(i, j) = a(i, j) − a(i, k) * a(k, j)
    Next j
  Next i
Next k

x(n) = a(n, n + 1) / a(n, n)          ‘後退代入
For k = n − 1 To 1 Step −1            ‘大きい方から始める
  x(k) = a(k, n + 1)
  For j = k + 1 To n
    x(k) = x(k) − a(k, j) * x(j)
  Next j
Next k

For i = 1 To n                        ‘解をセルに出力
  Cells(i + 2, n + 4) = x(i)
Next i
End Sub
```