重回帰分析の原理
（パラメータ2個）

平均値$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

求めたい回帰直線

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$L = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}\{y_i - (\beta_0 + \beta_1 x_i + \beta_2 x_{i2})\}^2$$

偏微分して0とする

$$\frac{\partial L}{\partial \beta_0} = -2\sum_{i=1}^{n}\{y_i - (\beta_0 + \beta_1 x_1 + \beta_2 x_{i2})\} = 0$$

$$\frac{\partial L}{\partial \beta_1} = -2\sum_{i=1}^{n} x_{i1}\{y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2})\} = 0$$

$$\frac{\partial L}{\partial \beta_2} = -2\sum_{i=1}^{n} x_{i2}\{y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2})\} = 0$$

β_0を消去します

$$\beta_1\sum_{i=1}^{n}(x_{i1} - \overline{x_1})^2 - \beta_2\sum_{i=1}^{n}(x_{i1} - \overline{x_1})(x_{i2} - \overline{x_2}) = \sum_{i=1}^{n}(x_{i1} - \overline{x_1})(y_i - \bar{y})$$

$$\beta_1\sum_{i=1}^{n}(x_{i1} - \overline{x_1})(x_{i2} - \overline{x_2}) - \beta_2\sum_{i=1}^{n}(x_{i2} - \overline{x_2})^2 = \sum_{i=1}^{n}(x_{i2} - \overline{x_2})(y_i - \bar{y})$$

$$S_{11}\beta_1 + S_{12}\beta_2 = S_{1y}$$

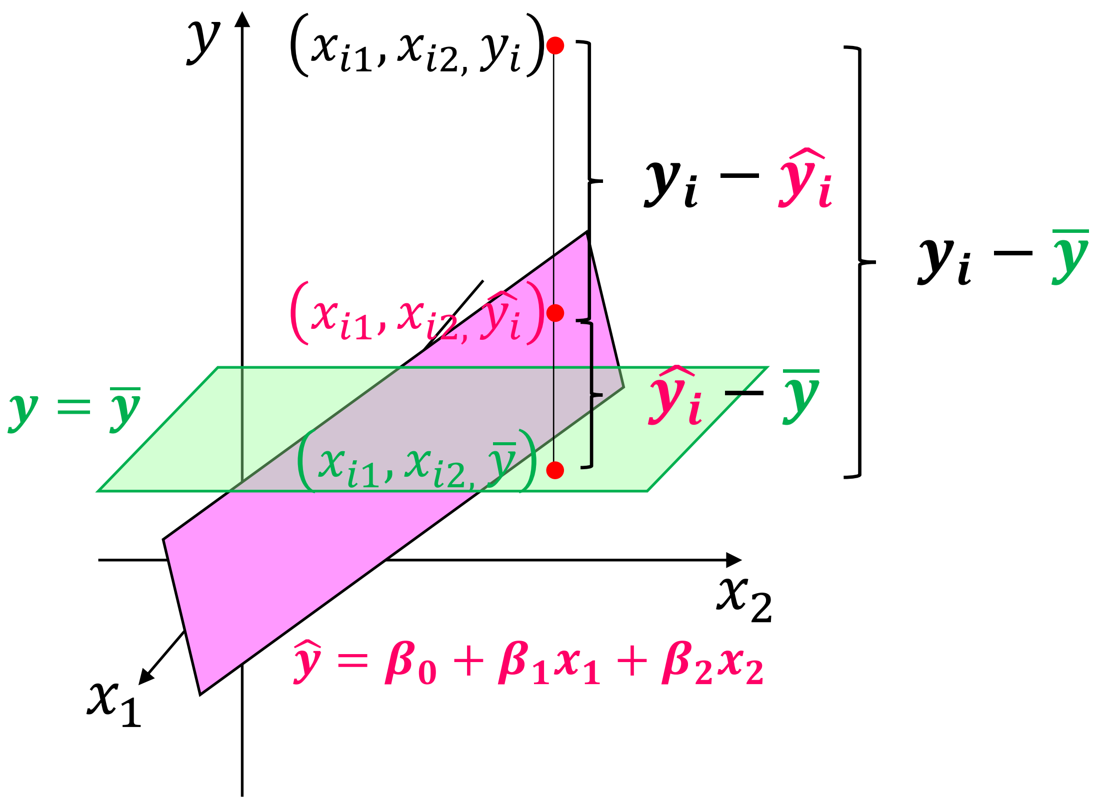
$$S_{21}\beta_1 + S_{22}\beta_2 = S_{2y}$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} S_{1y} \\ S_{2y} \end{pmatrix}$$

偏差平方和：$S_{jj} = \sum_{i=1}^{n}\left(x_{ij} - \overline{x_j}\right)^2 \quad (j = 1, 2)$

積和　　　：$S_{jk} = \sum_{i=1}^{n}\left(x_{ij} - \overline{x_j}\right)(x_{ik} - \overline{x_k}) \quad (j \neq k \quad j = 1, 2 \quad k = 1, 2)$

偏差積和　：$S_{jy} = \sum_{i=1}^{n}\left(x_{ij} - \overline{x_j}\right)(y_i - \bar{y}) \quad (j = 1,2)$



重回帰分析の原理
（パラメータ複数）

求めたい回帰直線

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots\cdots + \beta_p x_p$$

$$L = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}\{y_i - (\beta_0 + \beta_1 x_i + \beta_2 x_{i2} + \cdots\cdots + \beta_p x_{ip})\}^2$$

偏微分して0とする

$$\frac{\partial L}{\partial \beta_0} = -2\sum_{i=1}^{n}\{y_i - (\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots\cdots + \beta_p x_{ip})\} = 0$$

$$\frac{\partial L}{\partial \beta_1} = -2\sum_{i=1}^{n} x_{i1}\{y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots\cdots + \beta_p x_p)\} = 0$$

$$\vdots$$

$$\frac{\partial L}{\partial \beta_p} = -2\sum_{i=1}^{n} x_{ip}\{y_i - (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots\cdots + \beta_p x_{ip})\} = 0$$

偏差平方和： $S_{jj} = \sum_{i=1}^{n}(x_{ij} - \overline{x_j})^2 \quad (j = 1, 2 \cdots\cdots p)$

積和 ： $S_{jk} = \sum_{i=1}^{n}(x_{ij} - \overline{x_j})(x_{ik} - \overline{x_k}) \quad (j \neq k \quad j = 1, 2 \cdots p \quad k = 1, 2 \cdots p)$

偏差積和 ： $S_{jy} = \sum_{i=1}^{n}(x_{ij} - \overline{x_j})(y_i - \bar{y}) \quad (j = 1,2 \cdots p)$

β_0を消去します

$$S_{11}\beta_1 + S_{12}\beta_2 + \cdots\cdots + S_{1p}\beta_p = S_{1y}$$
$$S_{21}\beta_1 + S_{22}\beta_2 + \cdots\cdots + S_{2p}\beta_p = S_{2y}$$
$$\vdots$$
$$S_{n1}\beta_1 + S_{n2}\beta_2 + \cdots\cdots + S_{np}\beta_p = S_{py}$$

$$\begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{np} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} = \begin{pmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{py} \end{pmatrix}$$

$$S = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{np} \end{pmatrix} \qquad \vec{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} \qquad M = \begin{pmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{py} \end{pmatrix}$$

$$S\vec{\beta} = M \qquad \vec{\beta} = S^{-1}M$$

手作業の手順

求めたいのは重回帰式$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots\cdots + \beta_p x_p$**の係数**$\beta_0$**〜**$\beta_p$

①偏差平方和： $S_{jj} = \sum_{i=1}^{n}\left(x_{ij} - \bar{x_j}\right)^2 \qquad (j = 1, 2 \cdots 4)$

x_1〜x_4各列の平方和　関数は、SUMSQ（〇：〇）

重回帰分析

【課題】 重回帰式の係数を算出せよ

						S_{jj}			
						$\Sigma(x_{i1}-x_{1bar})^2$	$\Sigma(x_{i2}-x_{2bar})^2$	$\Sigma(x_{i3}-x_{3bar})^2$	$\Sigma(x_{i4}-x_{4bar})^2$
	n数	225							
	y	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
i	家賃	広さ	築年数	徒歩	方角	$x_{i1}-x_{1bar}$	$x_{i2}-x_{2bar}$	$x_{i3}-x_{3bar}$	$x_{i4}-x_{4bar}$
1	67.3	19.3	18	10	0				
2	70.3	24.0	5	20	0				
3	73.9	20.7	18	5	0				
4	74.4	20.0	12	15	1				
5	75.6	18.4	10	8	1				
≀									
224	342.6	107.4	1	5	0				
225	342.6	107.4	1	5	0				
平均	177.0	53.6	8.2	10.2	0.5				

x_1〜x_4のデータから
平均値を引く

②積和：$S_{jk}=\sum_{i=1}^{n}(x_{ij}-\overline{x_j})(x_{ik}-\overline{x_k})$

各列の積和の合計　関数は、SUM（○：○）

S_{jj}				S_{jk}					
$\Sigma(x_{i1}-x_{1bar})^2$	$\Sigma(x_{i2}-x_{2bar})^2$	$\Sigma(x_{i3}-x_{3bar})^2$	$\Sigma(x_{i4}-x_{4bar})^2$	$\Sigma(x_{i1}-x_{1bar})\times(x_{i2}-x_{2bar})$	$\Sigma(x_{i2}-x_{2bar})\times(x_{i3}-x_{3bar})$	$\Sigma(x_{i3}-x_{3bar})\times(x_{i4}-x_{4bar})$	$\Sigma(x_{i4}-x_{4bar})\times(x_{i1}-x_{1bar})$	$\Sigma(x_{i1}-x_{1bar})\times(x_{i3}-x_{3bar})$	$\Sigma(x_{i2}-x_{2bar})\times(x_{i4}-x_{4bar})$

x_1	x_2	x_3	x_4						
$x_{i1}-x_{1bar}$	$x_{i2}-x_{2bar}$	$x_{i3}-x_{3bar}$	$x_{i4}-x_{4bar}$	$(x_{i1}-x_{1bar})\times(x_{i2}-x_{2bar})$	$(x_{i2}-x_{2bar})\times(x_{i3}-x_{3bar})$	$(x_{i3}-x_{3bar})\times(x_{i4}-x_{4bar})$	$(x_{i4}-x_{4bar})\times(x_{i1}-x_{1bar})$	$(x_{i1}-x_{1bar})\times(x_{i3}-x_{3bar})$	$(x_{i2}-x_{2bar})\times(x_{i4}-x_{4bar})$

全ての列の積を計算

③偏差積和　$:S_{jy}=\sum_{i=1}^{n}(x_{ij}-\bar{x_j})(y_i-\bar{y})\quad (j=1,2\cdots 4)$

各列の偏差積和の合計　関数は、SUM(〇:〇)

S_{ii}				
$\Sigma(x_{i1}-x_{1bar})^2$	$\Sigma(x_{i2}-x_{2bar})^2$	$\Sigma(x_{i3}-x_{3bar})^2$	$\Sigma(x_{i4}-x_{4bar})^2$	$\Sigma(x$ $(x_{i2}$ -
x_1	x_2	x_3	x_4	
$x_{i1}-x_{1bar}$	$x_{i2}-x_{2bar}$	$x_{i3}-x_{3bar}$	$x_{i4}-x_{4bar}$	$(x_{i1}$ $(y$

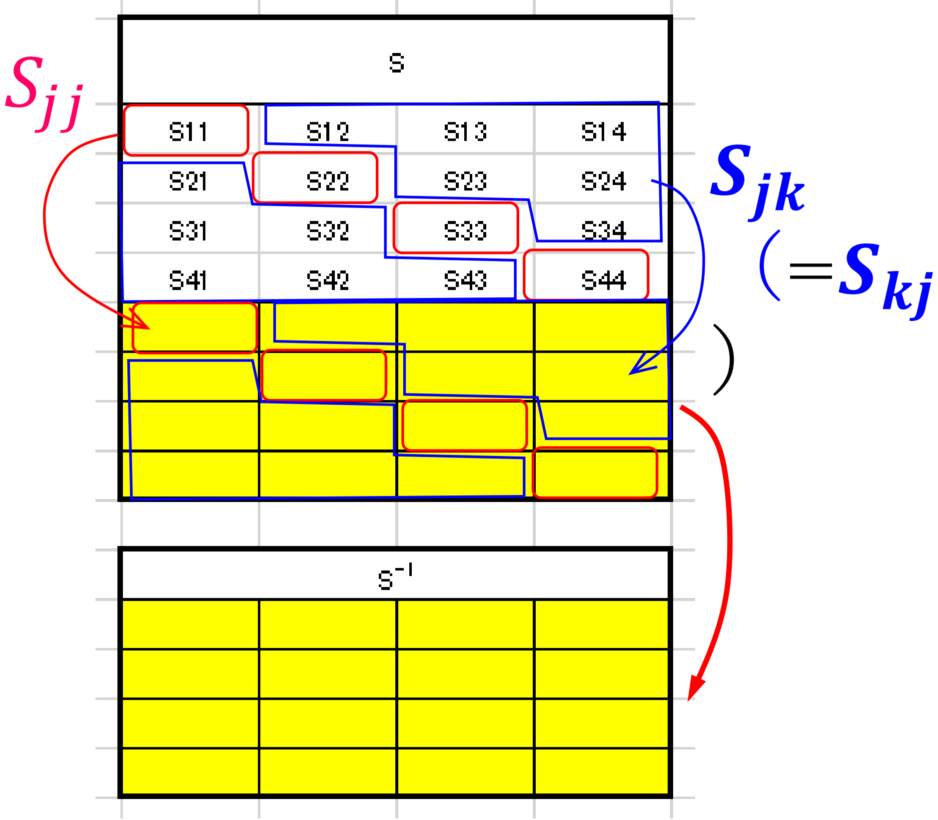
	S_{iy}			
	$\Sigma(x_{i1}-x_{1bar})\times(y_i-y_{bar})$	$\Sigma(x_{i2}-x_{2bar})\times(y_i-y_{bar})$	$\Sigma(x_{i3}-x_{3bar})\times(y_i-y_{bar})$	$\Sigma(x_{i4}-x_{4bar})\times(y_i-y_{bar})$
	0.0	0.0	0.0	0.0
y_i-y_{bar}	$(x_{i1}-x_{1bar})\times(y_i-y_{bar})$	$(x_{i2}-x_{2bar})\times(y_i-y_{bar})$	$(x_{i3}-x_{3bar})\times(y_i-y_{bar})$	$(x_{i4}-x_{4bar})\times(y_i-y_{bar})$

④求めたいのは重回帰式$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots\cdots + \beta_p x_p$の係数$\beta_0 \sim \beta_p$
行列Sの逆行列S−1を求め行列Mを掛けるとβ の行列が求まる

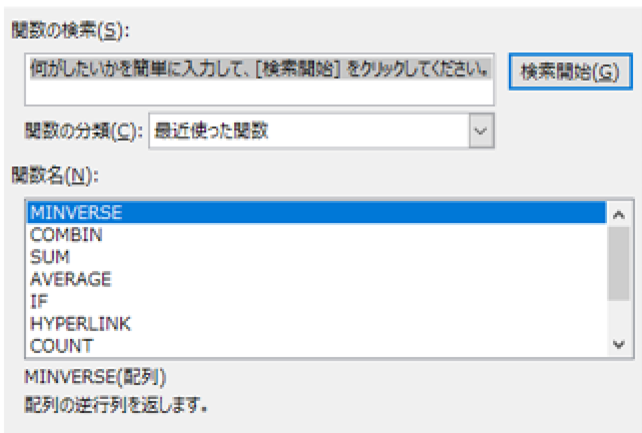
$$\begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{np} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} = \begin{pmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{py} \end{pmatrix}$$

$$S = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1p} \\ S_{21} & S_{22} & \cdots & S_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{np} \end{pmatrix} \quad \vec{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} \quad M = \begin{pmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{py} \end{pmatrix}$$

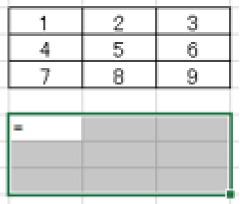
$$S\vec{\beta} = M \qquad \vec{\beta} = S^{-1}M$$

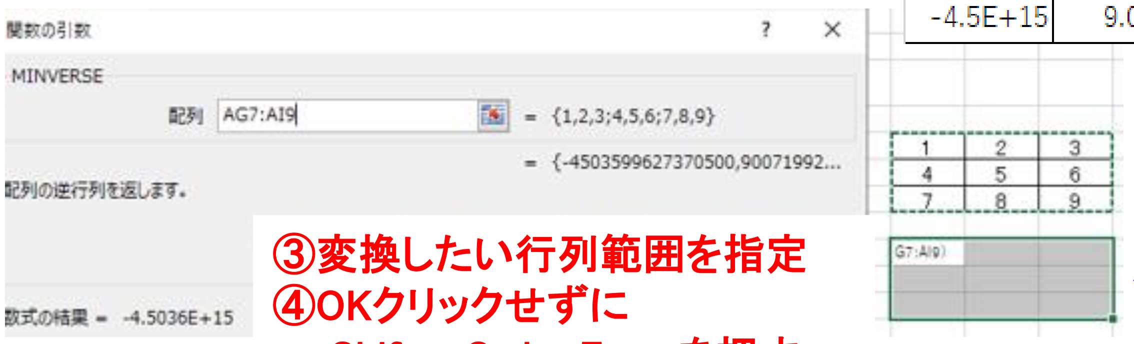


Excelで逆行列を求める方法



①逆行列の結果を入れる領域を指定
②数式　MINVERSEを選択





③変換したい行列範囲を指定
④OKクリックせずに
Shift + Ctrl + Enterを押す

実行結果

-4.5E+15	9.0E+15	-4.5E+15
9.0E+15	-1.8E+16	9.0E+15
-4.5E+15	9.0E+15	-4.5E+15

Siy			
Σ(xi1 − x1bar)×(yi−ybar)	Σ(xi2 − x2bar)×(yi− ybar)	Σ(xi3 − x3bar)×(yi− ybar)	Σ(xi4 − x4bar)×(yi− ybar)

$$M = \begin{pmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{py} \end{pmatrix}$$

$$\vec{\beta} = S^{-1}M$$

S^{-1}				M		β
4.6E−06	4.0E−07	3.8E−06	−1.0E−06	643164.8	1	2.87E+00
4.0E−07	1.6E−04	9.8E−06	−1.0E−05	−7939.6	2	−1.12E+00
3.8E−06	9.8E−06	2.3E−04	−1.2E−05	−13723.6	3	−8.22E−01
−1.0E−06	−1.0E−05	−1.2E−05	1.8E−02	104.4	4	1.45E+00

	β_0
0	3.97E+01

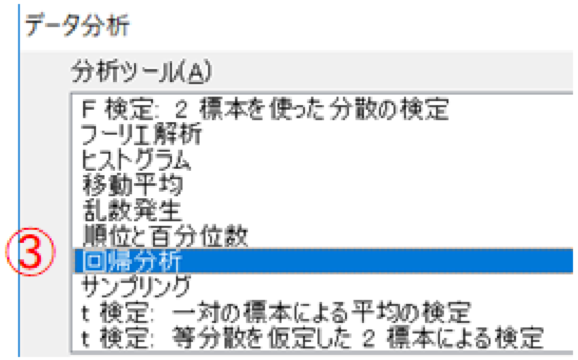
	y	x_1	x_2	x_3	x_4
i	家賃	広さ	築年数	徒歩	方角
1	67.3	19.3	18	10	0
2	70.3	24.0	5	20	0
3	73.9	20.7	18	5	0
4	74.4	20.0	12	15	1
≀					
221	340.7	107.2	1	8	0
222	341.1	109.5	3	15	0
223	342.0	110.4	5	15	1
224	342.6	107.4	1	5	0
225	342.6	107.4	1	5	0
平均	177.0	53.6	8.2	10.2	0.5
				全平均	41.6

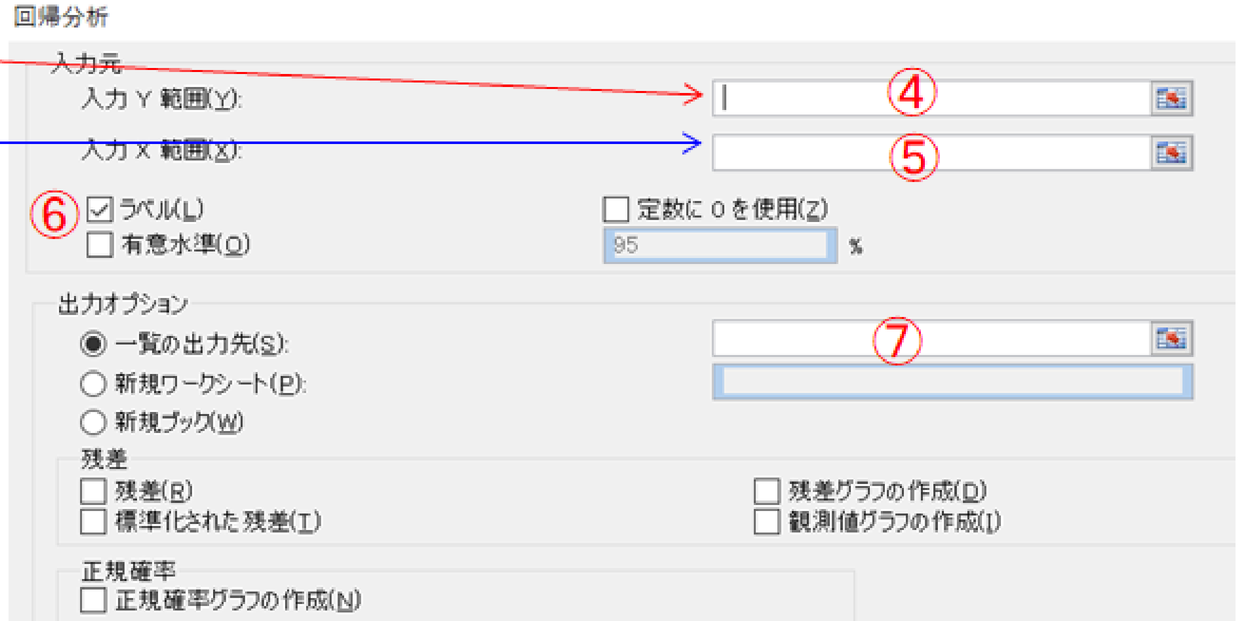
$$\beta_0 = \hat{y} - (\beta_1 x_1 + \beta_2 x_2 + \cdots\cdots + \beta_4 x_4)$$

Excelの重回帰分析の手順

y　　x

家賃	広さ	築年数	徒歩	方角
67.3	19.3	18	10	0
70.3	24.0	5	20	0
73.9	20.7	18	5	0
74.4	20.0	12	15	1
75.6	18.4	10	8	1
76.0	19.0	10	10	0
77.7	18.7	10	5	0
78.4	22.8	18	8	0
79.3	22.9	18	8	1
79.6	19.5	8	10	0
81.0	18.8	3	15	0
81.2	21.3	12	8	0
82.0	21.8	12	10	1
83.5	20.5	5	15	0





手作業の実行結果

	β
1	2.87E+00
2	-1.12E+00
3	-8.22E-01
4	1.45E+00

	β_0
0	3.97E+01

分析ツールの実行結果

概要

回帰統計	
重相関 R	0.999236
重決定 R2	0.998472
補正 R2	0.998444
標準誤差	3.604357
観測数	225

分散分析表

	自由度	変動	分散	刂された分背	有意 F
回帰	4	1867779	466944.7	35942.63	0
残差	220	2858.106	12.99139		
合計	224	1870637			

	係数	標準誤差	t	P-値	下限 95%	上限 95%	下限 95.0%	上限 95.0%
切片	39.71026	0.899141	44.16465	2.5E-111	37.93823	41.48229	37.93823	41.48229
広さ	2.872509	0.00769	373.5612	0	2.857355	2.887664	2.857355	2.887664
築年数	-1.11548	0.045024	-24.7755	1.39E-65	-1.20421	-1.02675	-1.20421	-1.02675
徒歩	-0.82152	0.054645	-15.0339	1.3E-35	-0.92922	-0.71383	-0.92922	-0.71383
方角	1.44547	0.480605	3.007607	0.00294	0.498292	2.392649	0.498292	2.392649