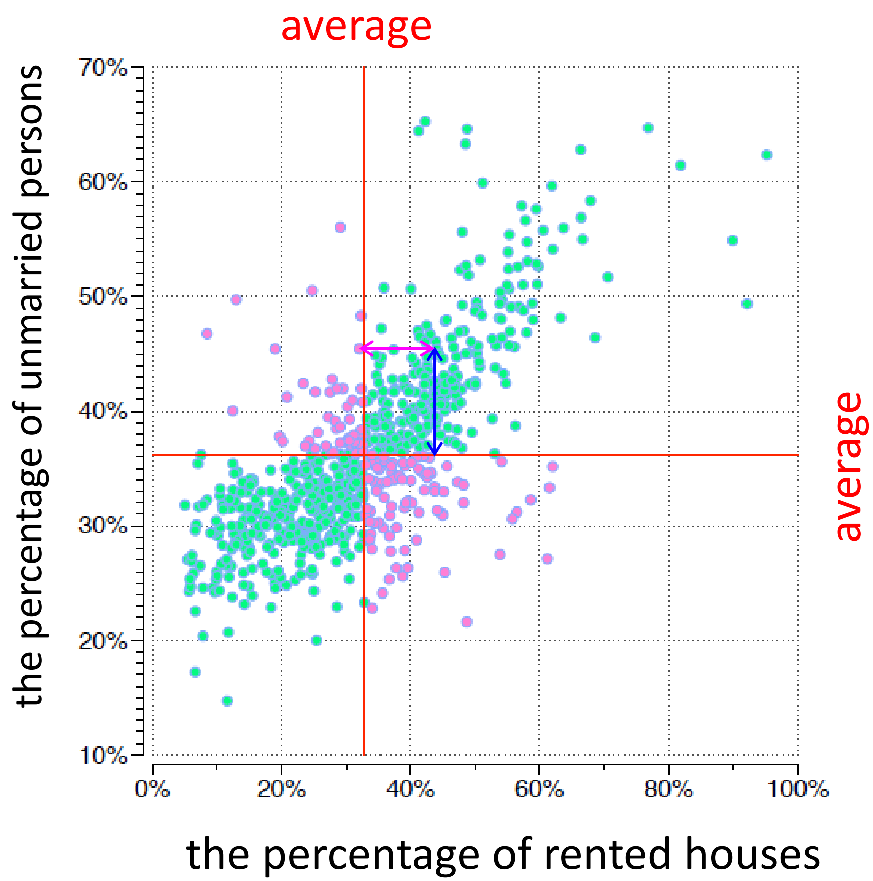
Ｑ２　What is orthogonal?

correlation coefficient

covariance

$(x_i - \bar{x})((y_j - \bar{y})$

	x	$x_i - \bar{x}$	y	$y_j - \bar{y}$	
a	43.9	11.0	46.1	9.9	108.9
b	31.7	-1.2	36.6	0.4	-0.5
c	29.5	-3.4	42.0	5.8	-19.7
…	40.5	7.6	40.3	4.1	31.2
…	32.4	-0.5	37.9	1.6	-0.8
…	59.0	26.1	48.0	11.8	308.0
…	46.6	13.8	43.7	7.5	103.5
…	28.4	-4.5	31.5	-4.7	21.2
…	28.7	-4.2	32.1	-4.1	17.2
…	29.5	-3.4	32.0	-4.2	14.3
…	35.9	3.0	32.5	-3.7	-11.1
…	32.6	-0.3	33.0	-3.2	1.0
…	38.6	5.7	38.0	1.8	10.3
…	31.0	-1.9	30.8	-5.4	10.3
…	18.5	-14.3	25.7	-10.5	150.2
…	45.3	12.4	40.0	3.8	47.1
…	25.2	-7.7	36.7	0.5	-3.9
…	50.3	17.4	45.0	8.8	153.1
…	54.7	21.9	42.4	6.2	135.8
…	30.0	-2.8	32.7	-3.5	9.8
…	50.2	17.4	49.5	13.3	231.4
…	44.9	12.0	41.6	5.4	64.8
…	27.6	-5.2	29.2	-7.0	36.4
…	50.0	17.1	42.6	6.4	109.4
…	47.3	14.4	40.6	4.4	63.4
…	…	…	…	…	…
				Average	84.3



covariance $\sum (x_i - \bar{x})(y_j - \bar{y})$

correlation coefficient

$$\textbf{covariance} = \frac{84.3}{14.4 \times 8.1} = 0.723$$

covariance of x

covariance of y

correlation coefficient

$$\frac{\sum (x_i - \bar{x})(y_j - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2}\sqrt{\sum (y_j - \bar{y})^2}}$$

correlation coefficient

$-1 \leqq$ correlation coefficient $r \leqq 1$

correlation coefficient r vector X' Y' inner product

$$= \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2}\sqrt{\sum(y_i - \bar{y})^2}}$$

the lengths of the vectors X’ and Y’

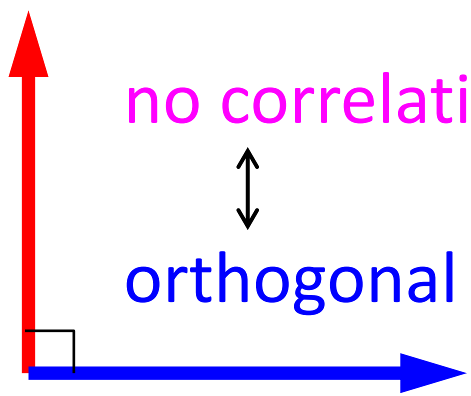
$$= \frac{X' \cdot Y'}{\|X'\|\|Y'\|}$$

$$= \cos\theta$$

$$X' = \begin{pmatrix} x_1 - \bar{x} \\ x_2 - \bar{x} \\ x_3 - \bar{x} \\ x_4 - \bar{x} \\ x_5 - \bar{x} \\ x_6 - \bar{x} \\ x_7 - \bar{x} \\ x_8 - \bar{x} \\ \vdots \\ x_{N\text{-}3} - \bar{x} \\ x_{N\text{-}3} - \bar{x} \\ x_{N\text{-}2} - \bar{x} \\ x_{N\text{-}1} - \bar{x} \\ x_N - \bar{x} \end{pmatrix} \quad Y' = \begin{pmatrix} y_1 - \bar{y} \\ y_2 - \bar{y} \\ y_3 - \bar{y} \\ y_4 - \bar{y} \\ y_5 - \bar{y} \\ y_6 - \bar{y} \\ y_7 - \bar{y} \\ y_8 - \bar{y} \\ \vdots \\ y_{N\text{-}3} - \bar{y} \\ y_{N\text{-}3} - \bar{y} \\ y_{N\text{-}2} - \bar{y} \\ y_{N\text{-}1} - \bar{y} \\ y_N - \bar{y} \end{pmatrix}$$

$\theta=0°$ $\cos\theta=1$ $r=1$

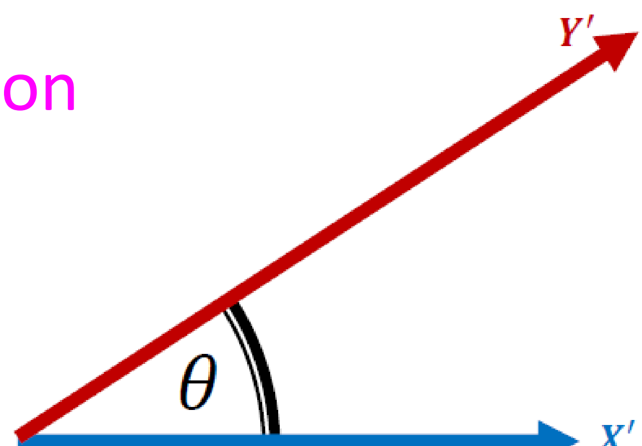
$\theta=90°$ $\cos\theta=0$ $r=0$



no correlation

↕

orthogonal



$$\text{correlation coefficient} = \frac{\text{covariance}}{\text{covariance}1 \times \text{covariance}2}$$

	A	B	C	D
Ex.1	10	−2	−3	1
Ex.2	10	5	−5	−1
Ex.3	10	−3	8	0
Ex.4	−6	−2	−5	0
Ex.5	−6	5	8	1
Ex.6	−6	−3	−3	−1
Ex.7	−4	−2	8	−1
Ex.8	−4	5	−3	0
Ex.9	−4	−3	−5	1

control factor

	A	B	C	D	SN比
Ex.1	1	1	1	1	
Ex.2	1	2	2	2	
Ex.3	1	3	3	3	
Ex.4	2	1	2	3	
Ex.5	2	2	3	1	
Ex.6	2	3	1	2	
Ex.7	3	1	3	2	
Ex.8	3	2	1	3	
Ex.9	3	3	2	1	

	A	B	C	D
level1	10	−2	−3	1
level2	−6	5	−5	−1
level1	−4	−3	8	0

	B×C	C×D	B×D	D×A	A×B	A×C
covariance	0	0	0	0	0	0
covariance1	3.6	5.7	3.6	0.8	7.1	7.1
covariance2	5.7	0.8	0.8	7.1	3.6	5.7
correlation coefficient	0	0	0	0	0	0

When focusing on the effect of control factor A
The correlation coefficients of the columns B and C, C and D, and B and D are all zero.

⇩

The columns of B, C and D are orthogonal to each other