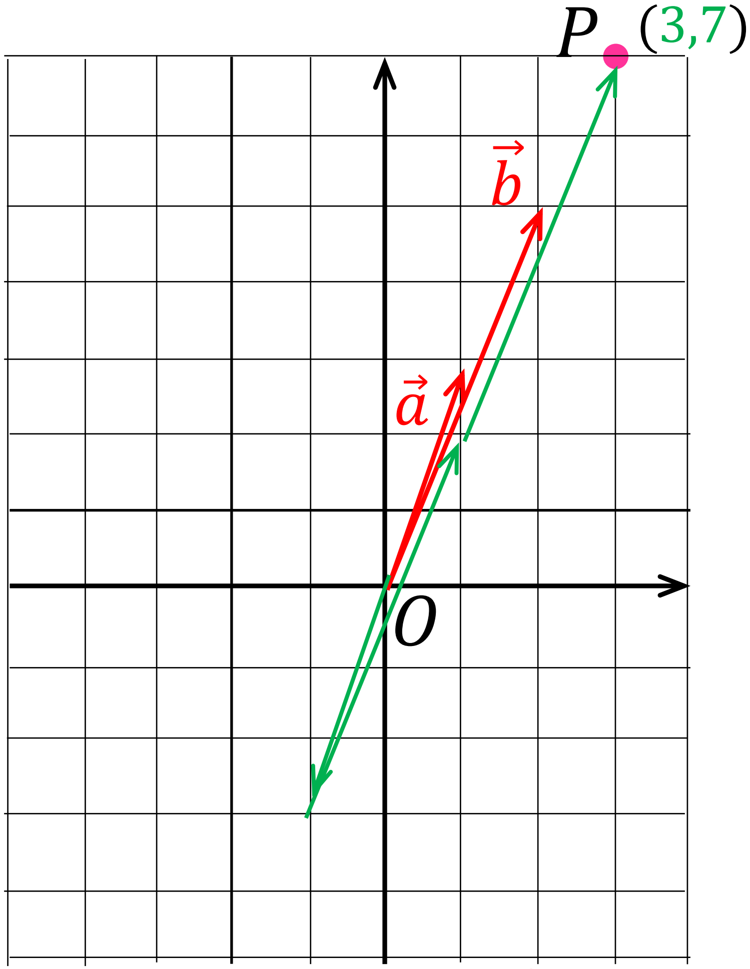
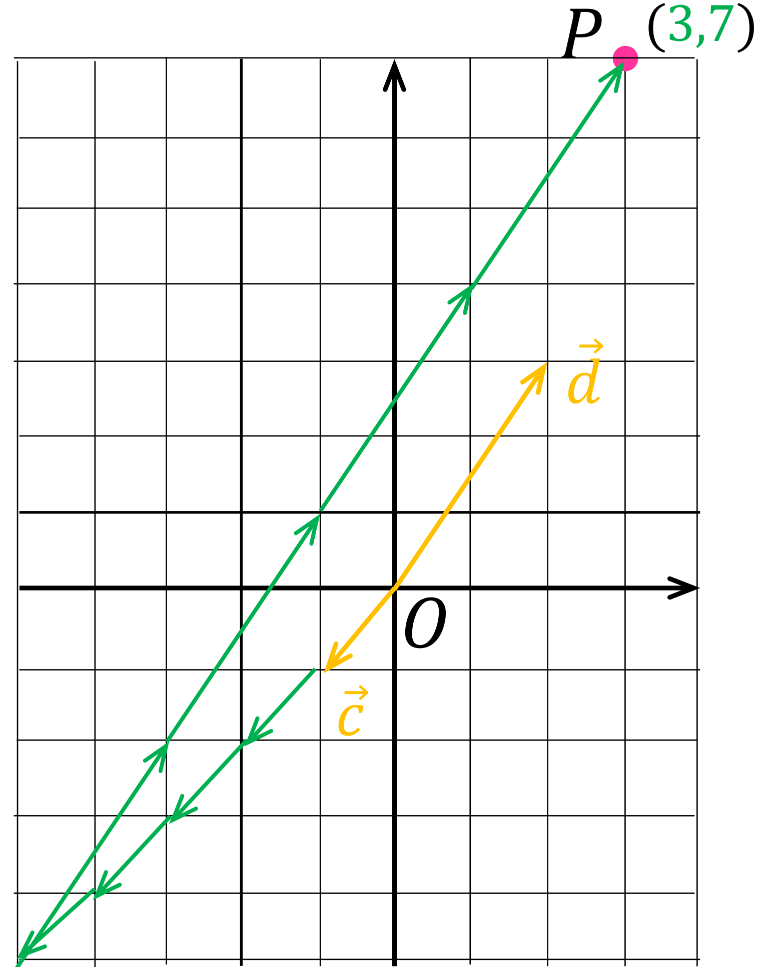




$\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$k\vec{a} + l\vec{b} = k\begin{pmatrix} 1 \\ 3 \end{pmatrix} + l\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} k+2l \\ 3k+5l \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$より$k + 2l = 3$　$3k + 5l = 7$

$k = -1$　$l = 2$

$$(-1)\vec{a} + 2\vec{b} = -\begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} = 5\begin{pmatrix} -1 \\ -1 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 5\vec{c} + 4\vec{d}$$

$$\vec{a}=\begin{pmatrix}1\\3\end{pmatrix}\qquad \vec{b}=\begin{pmatrix}2\\5\end{pmatrix}\qquad \vec{c}=\begin{pmatrix}-1\\-1\end{pmatrix}\qquad \vec{d}=\begin{pmatrix}2\\3\end{pmatrix}$$

$\vec{c}=\begin{pmatrix}-1\\-1\end{pmatrix}=k\vec{a}+l\vec{b}=k\begin{pmatrix}1\\3\end{pmatrix}+l\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}k+2l\\3k+5l\end{pmatrix}$より$k+2l=-1\quad 3k+5l=-1$

$k=3\quad l=-2$

$\vec{d}=\begin{pmatrix}2\\3\end{pmatrix}=m\vec{a}+n\vec{b}=m\begin{pmatrix}1\\3\end{pmatrix}+n\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}m+2n\\3m+5n\end{pmatrix}$より$m+2n=2\ 3m+5n=3$

$m=-4\quad n=3$

$$\vec{c}=\begin{pmatrix}-1\\-1\end{pmatrix}=3\begin{pmatrix}1\\3\end{pmatrix}-2\begin{pmatrix}2\\5\end{pmatrix}=(\vec{a}\ \vec{b})\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}1&2\\3&5\end{pmatrix}\begin{pmatrix}3\\-2\end{pmatrix}$$

$$\vec{d}=\begin{pmatrix}2\\3\end{pmatrix}=-4\begin{pmatrix}1\\3\end{pmatrix}+3\begin{pmatrix}2\\5\end{pmatrix}=(\vec{a}\ \vec{b})\begin{pmatrix}-4\\3\end{pmatrix}=\begin{pmatrix}1&2\\3&5\end{pmatrix}\begin{pmatrix}-4\\3\end{pmatrix}$$

合わせて$\begin{pmatrix}-1&2\\-1&3\end{pmatrix}=\begin{pmatrix}1&2\\3&5\end{pmatrix}\begin{pmatrix}3&-4\\-2&3\end{pmatrix}$

$$(\vec{c}\ \vec{d})=(\vec{a}\ \vec{b})\begin{pmatrix}3&-4\\-2&3\end{pmatrix}$$

$(\vec{c}\ \vec{d})=(\vec{a}\ \vec{b})P$ ←$(\vec{a}\ \vec{b})$から$(\vec{c}\ \vec{d})$への

基底の取替行列

$\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ の時、$\begin{pmatrix} 3 \\ 7 \end{pmatrix}$は$(-1)\vec{a} + 2\vec{b}$で表せる

$\vec{c} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$では、$\begin{pmatrix} 3 \\ 7 \end{pmatrix}$を$k\vec{c} + l\vec{d}$ で表したい kとlは？

$$(-1)\vec{a} + 2\vec{b} = k\overrightarrow{\mathrm{c}} + l\vec{\mathrm{d}}$$

$$\begin{pmatrix} \vec{a} & \vec{b} \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} \vec{c} & \vec{d} \end{pmatrix}\begin{pmatrix} k \\ l \end{pmatrix} = \begin{pmatrix} \vec{a} & \vec{b} \end{pmatrix}P\begin{pmatrix} k \\ l \end{pmatrix} \longleftarrow \begin{pmatrix} \vec{c} & \vec{d} \end{pmatrix} = \begin{pmatrix} \vec{a} & \vec{b} \end{pmatrix}P$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} = P\begin{pmatrix} k \\ l \end{pmatrix}$$

$P = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$の逆行列 $P^{-1} = \frac{1}{3 \cdot 3 - (-2) \cdot (-4)}\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$を**両辺に掛ける**

$$P^{-1}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = P^{-1}P\begin{pmatrix} k \\ l \end{pmatrix} = \boldsymbol{E}\begin{pmatrix} k \\ l \end{pmatrix} = \begin{pmatrix} k \\ l \end{pmatrix}$$

$$\begin{pmatrix} k \\ l \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$(-1)\vec{a} + 2\vec{b} = -\begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} = 5\begin{pmatrix} -1 \\ -1 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 5\vec{c} + 4\vec{d}$$

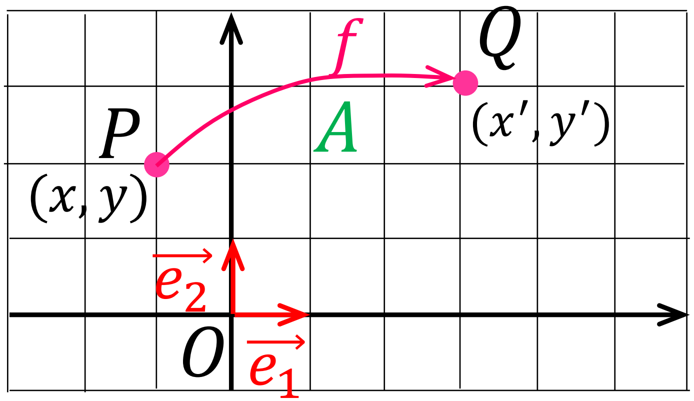
$(\vec{a}\ \ \vec{b})$から$(\vec{c}\ \ \vec{d})$への基底の取替行列P

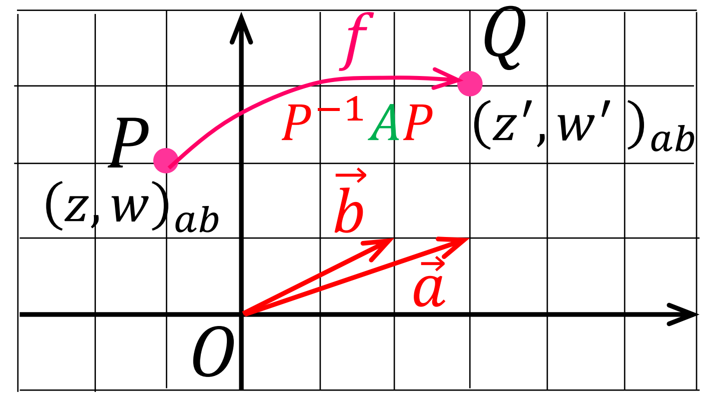
$$(\vec{c}\ \ \vec{d}) = (\vec{a}\ \ \vec{b})P$$

$$x\vec{a} + y\vec{b} = k\overrightarrow{\mathrm{c}} + l\vec{\mathrm{d}}$$

$(x, y)\ _{ab} = (k, l)\ _{cd}$を求めるには

$$P^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k \\ l \end{pmatrix}$$





$$\vec{a} = \begin{pmatrix}3\\1\end{pmatrix} \qquad \vec{b} = \begin{pmatrix}2\\1\end{pmatrix}$$

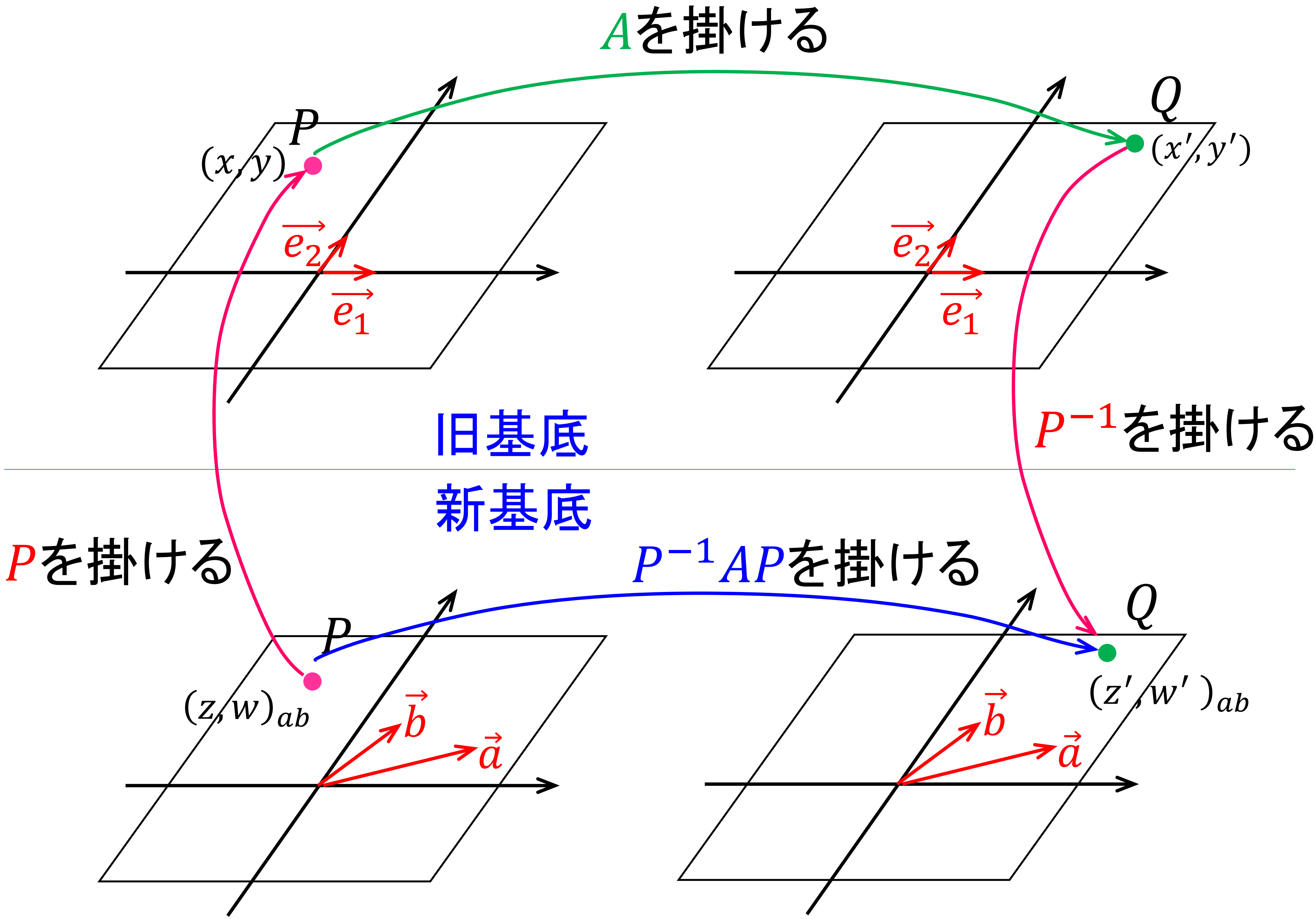
$\overrightarrow{e_1}$ $\overrightarrow{e_1}$に取替行列$P = \begin{pmatrix}3&2\\1&1\end{pmatrix}$を掛けて$\vec{a}$ $\vec{b}$にする式が

$$\begin{pmatrix}\vec{a} & \vec{b}\end{pmatrix} = \begin{pmatrix}\overrightarrow{e_1} & \overrightarrow{e_1}\end{pmatrix}\begin{pmatrix}3&2\\1&1\end{pmatrix} \qquad \begin{pmatrix}3&2\\1&1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}3&2\\1&1\end{pmatrix}$$

$$P^{-1}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}z\\w\end{pmatrix} \longrightarrow PP^{-1}\begin{pmatrix}x\\y\end{pmatrix} = P\begin{pmatrix}z\\w\end{pmatrix} \longrightarrow \begin{pmatrix}x\\y\end{pmatrix} = P\begin{pmatrix}z\\w\end{pmatrix}$$

$$P^{-1}\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}z'\\w'\end{pmatrix} \qquad \begin{pmatrix}x'\\y'\end{pmatrix} = P\begin{pmatrix}z'\\w'\end{pmatrix}$$

$\begin{pmatrix}x'\\y'\end{pmatrix} = A\begin{pmatrix}x\\y\end{pmatrix}$の時 $\quad \begin{pmatrix}z'\\w'\end{pmatrix} = P^{-1}\begin{pmatrix}x'\\y'\end{pmatrix} = P^{-1}A\begin{pmatrix}x\\y\end{pmatrix} = P^{-1}AP\begin{pmatrix}z\\w\end{pmatrix}$



Aを掛ける
P
(x, y)
Q
(x', y')
$\overrightarrow{e_2}$
$\overrightarrow{e_1}$
$\overrightarrow{e_2}$
$\overrightarrow{e_1}$
旧基底
P^{-1}を掛ける
新基底
Pを掛ける
$P^{-1}AP$を掛ける
P
$(z, w)_{ab}$
Q
$(z', w')_{ab}$
$\vec{b}$
$\vec{a}$
$\vec{b}$
$\vec{a}$