

対称行列

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 4 & 3 & -1 \\ -2 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 4 & -2 \\ 4 & 3 & -1 \\ -2 & -1 & 2 \end{pmatrix} = {}^t A$$

転置行列

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix} \quad P = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{2 \cdot 2 - (-1 \cdot 1)} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$P^{-1}AP = \frac{1}{5} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 & -2 \\ 7 & 14 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 10 & 0 \\ 0 & 35 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}$$

標準基底の表現行列

$$P = (\vec{p} \ \vec{q}) = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \quad \vec{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \vec{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{を大きさ1のベクトルにします}$$

$$\vec{p}^1 = \frac{1}{\sqrt{2^2 + (-1)^2}} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \vec{q}^1 = \frac{1}{\sqrt{1^2 + 2^2}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{より} \quad Q = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$Q^{-1} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} = {}^t Q$$

結果は同じ
逆行列の計算が容易

正規直交基底の表現行列

$$\begin{aligned} Q^{-1}AQ &= \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \\ &= \frac{1}{5} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 & -2 \\ 7 & 14 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \\ &= \frac{1}{5} \begin{pmatrix} 10 & 0 \\ 0 & 35 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix} \end{aligned}$$

$U = (\overrightarrow{u_1} \ \overrightarrow{u_2} \ \overrightarrow{u_3})$ 正規直交基底で構成

$$\begin{array}{lll} \overrightarrow{u_1} \cdot \overrightarrow{u_1} = 1 & \overrightarrow{u_2} \cdot \overrightarrow{u_2} = 1 & \overrightarrow{u_3} \cdot \overrightarrow{u_3} = 1 \\ \overrightarrow{u_1} \cdot \overrightarrow{u_2} = 0 & \overrightarrow{u_2} \cdot \overrightarrow{u_3} = 0 & \overrightarrow{u_3} \cdot \overrightarrow{u_1} = 0 \end{array}$$

$$\overrightarrow{u_1} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \quad \overrightarrow{u_2} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \overrightarrow{u_3} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\begin{array}{lll} A^2 + B^2 + C^2 = 1 & x^2 + y^2 + z^2 = 1 & a^2 + b^2 + c^2 = 1 \\ Ax + By + Cz = 0 & xa + yb + zc = 0 & aA + bB + cC = 0 \end{array}$$

$$U = \begin{pmatrix} A & x & a \\ B & y & b \\ C & z & c \end{pmatrix} \quad \text{直交行列}$$

$$\begin{aligned} U^{-1}U &= \begin{pmatrix} A & B & C \\ x & y & z \\ a & b & c \end{pmatrix} \begin{pmatrix} A & x & a \\ B & y & b \\ C & z & c \end{pmatrix} \\ &= \begin{pmatrix} A^2 + B^2 + C^2 & Ax + By + Cz & aA + bB + cC \\ Ax + By + Cz & x^2 + y^2 + z^2 & xa + yb + zc \\ aA + bB + cC & xa + yb + zc & a^2 + b^2 + c^2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{対角化} \end{aligned}$$