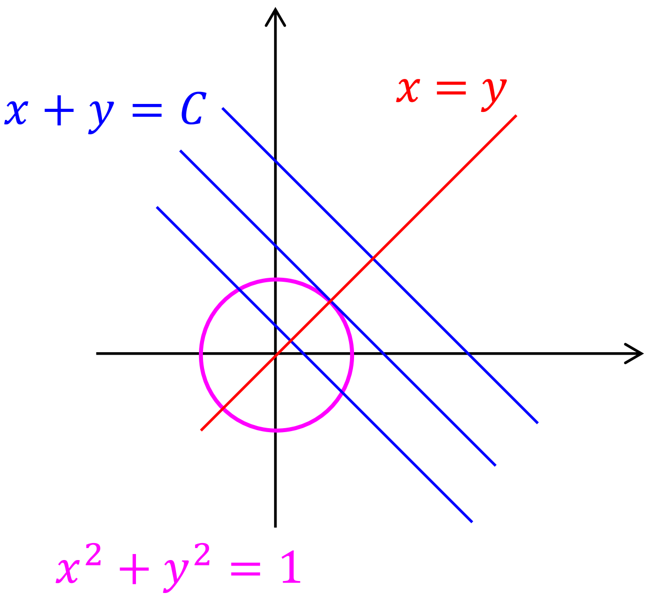


$x^2 + y^2 = 1$の条件の下
$f(x, y) = x + y$の最大値を求める

$x + y = C$に直交して円の中心を通る
直線は$x = y$
この直線と円の交点は
$x^2 + x^2 = 1$より　$x = \pm\frac{\sqrt{2}}{2}$　　$y = \pm\frac{\sqrt{2}}{2}$
$f(x, y)$の最大値は $x + y = \sqrt{2}$

ラグランジュの未定乗数法

$f(x,y)$の最大値を$g(x,y)=0$の制約条件の下に算出する

手順

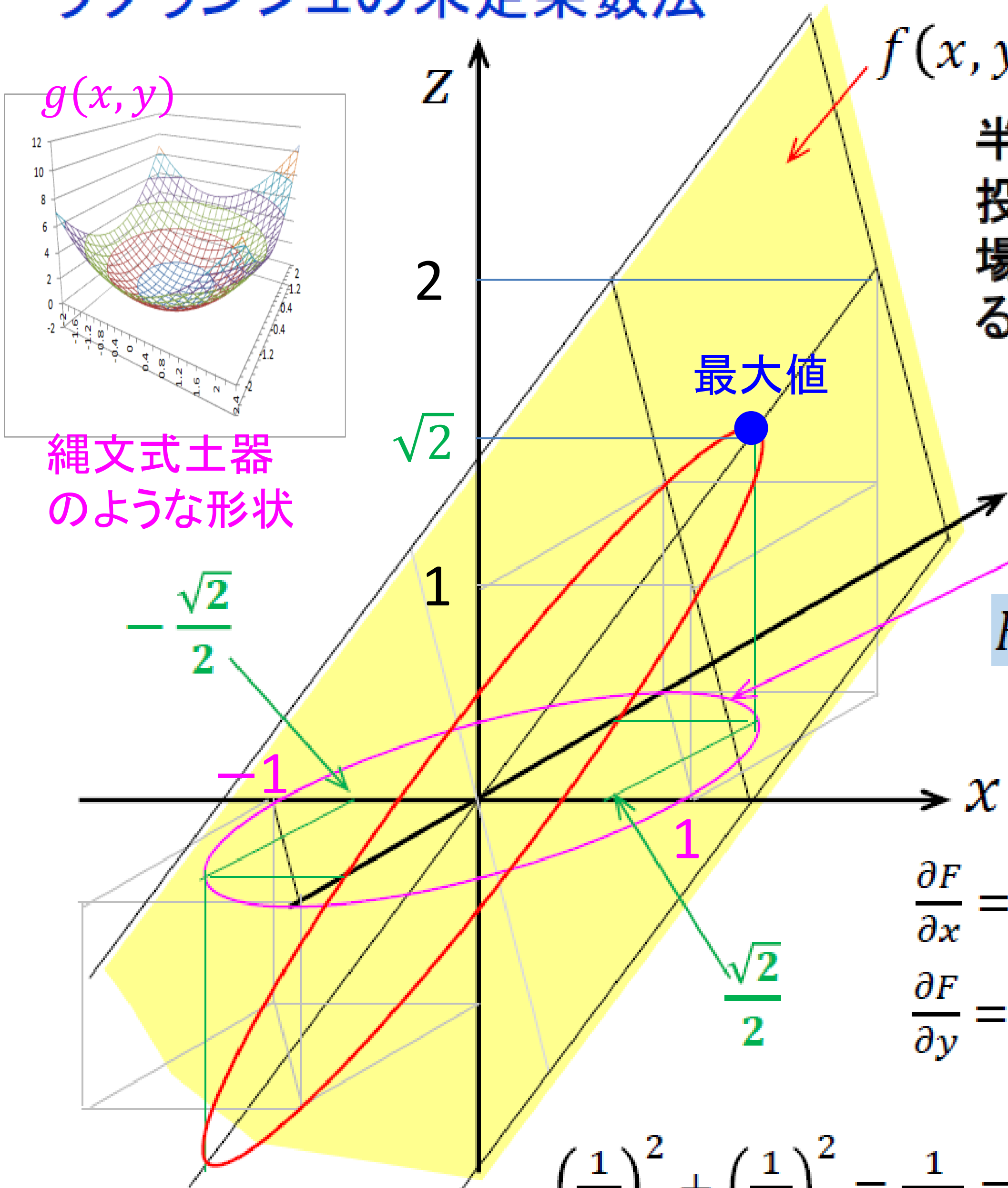
① $\boldsymbol{F(\lambda,x,y)=f(x,y)-\lambda g(\lambda,x,y)}$とおく

② $\boldsymbol{\frac{\partial F}{\partial \lambda}=\frac{\partial F}{\partial x}=\frac{\partial F}{\partial y}=0}$

③ $\boldsymbol{\lambda}$を求める

④ $\boldsymbol{x}$及び$\boldsymbol{y}$を求める

ラグランジュの未定乗数法



$f(x,y) = x + y$

半径1の円$g(x,y)$を平面$f(x,y)$に投影させた円上だけ移動できるとした場合、**最大**あるいは**最小**のz値を与えるx及びyを求める

$g(x,y) = x^2 + y^2 - 1 = 0$

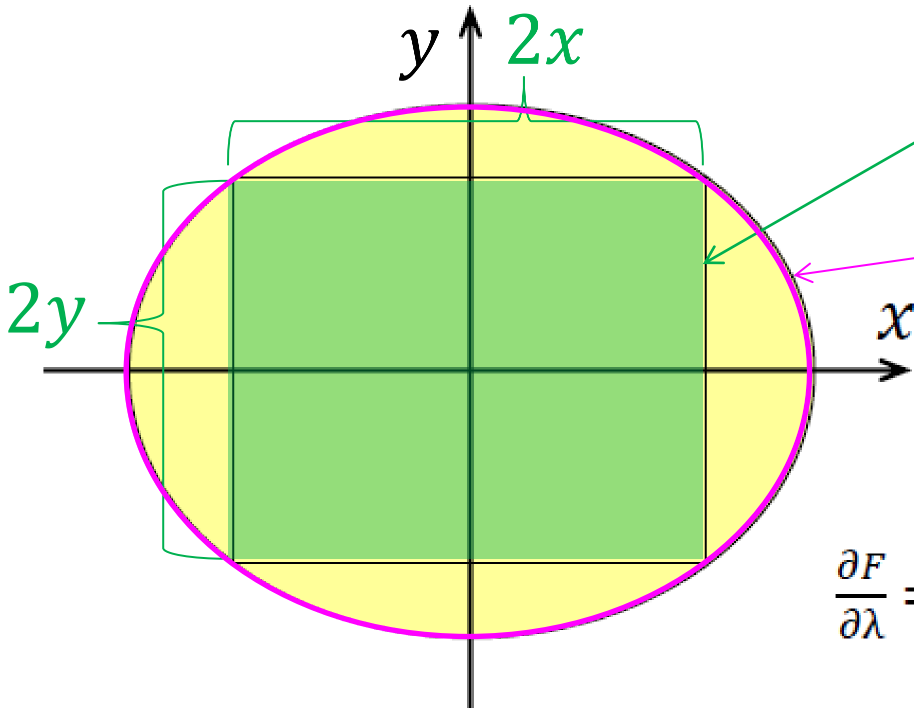
$$F(\lambda,x,y) = f(x,y) - \lambda g(x,y)$$

$$\frac{\partial F}{\partial \lambda} = -g(x,y)$$

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x} - \lambda\frac{\partial g}{\partial x} = 1 - 2\lambda x = 0 \qquad x = \frac{1}{2\lambda}$$

$$\frac{\partial F}{\partial y} = \frac{\partial f}{\partial y} - \lambda\frac{\partial g}{\partial y} = 1 - 2\lambda y = 0 \qquad y = \frac{1}{2\lambda}$$

$$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = \frac{1}{2\lambda^2} = 1$$ より $\lambda = \pm\frac{\sqrt{2}}{2}$ $\qquad$ $x = y = \pm\frac{\sqrt{2}}{2}$



$f(x,y) = 4xy$ ←最大の面積は？

$g(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

$F(\lambda, x, y) = f(x,y) - \lambda g(x,y)$

$\frac{\partial F}{\partial \lambda} = -\,g(x,y)$

$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} = 4y - \frac{2}{a^2}\lambda x = 0$ $\quad x = \frac{\lambda y}{2b^2}$ $\quad y = \frac{\lambda x}{2a^2}$

$\frac{\partial F}{\partial y} = \frac{\partial f}{\partial y} - \lambda \frac{\partial g}{\partial y} = 4x - \frac{2}{b^2}\lambda y = 0$ $\quad 4x - \frac{2}{b^2}\lambda \frac{\lambda x}{2a^2} = 0$ $\quad \left(4 - \frac{\lambda^2}{a^2 b^2}\right)x = 0$

$4 - \frac{\lambda^2}{a^2b^2} = 0$ より $\quad \lambda = \pm\sqrt{4a^2b^2} = \pm 2ab$

$\frac{x^2}{a^2} + \frac{1}{b^2}\left(\frac{\lambda x}{2a^2}\right)^2 - 1 = 0$ $\quad x = \pm\frac{\sqrt{2}a}{2}$ $\quad y = \pm\frac{\sqrt{2}b}{2}$