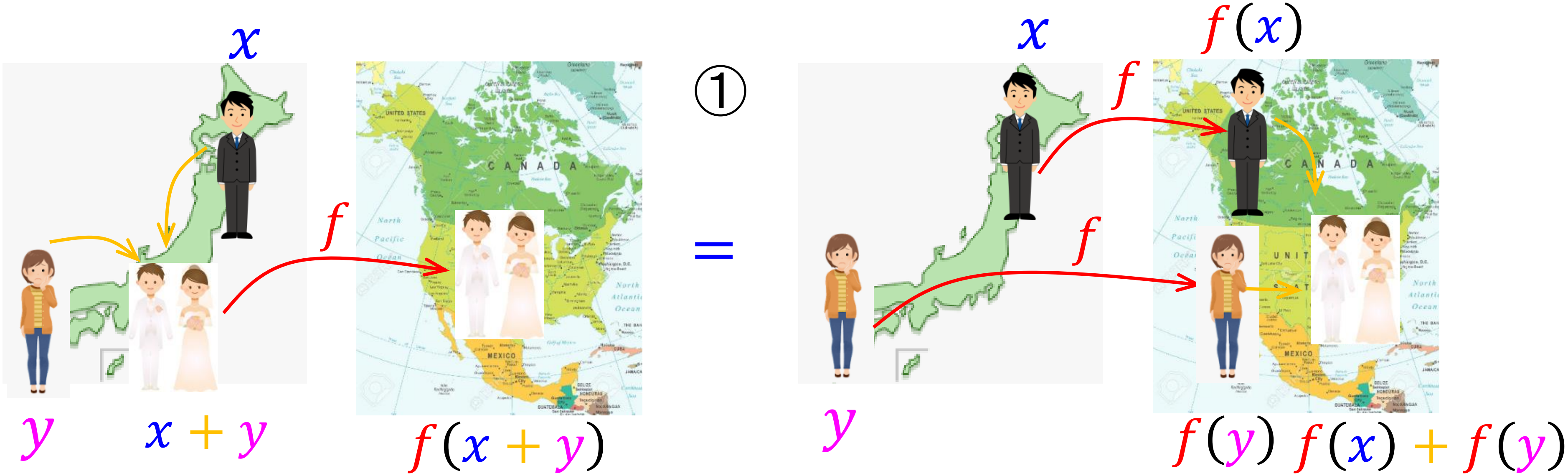
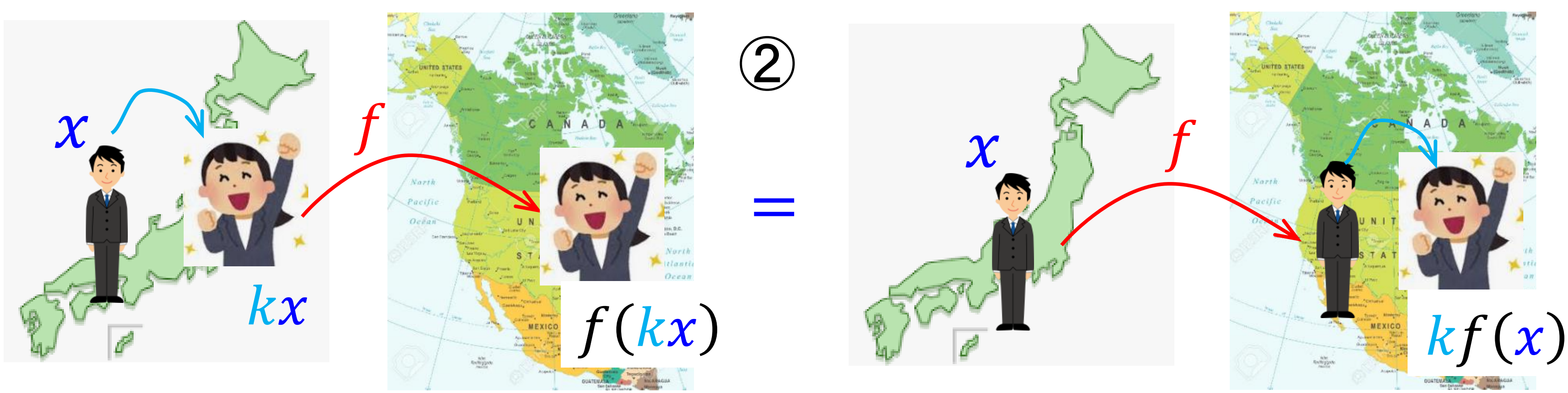
出典:「意味がわかる線形代数」(石井俊全　　発行所:ベル出版)

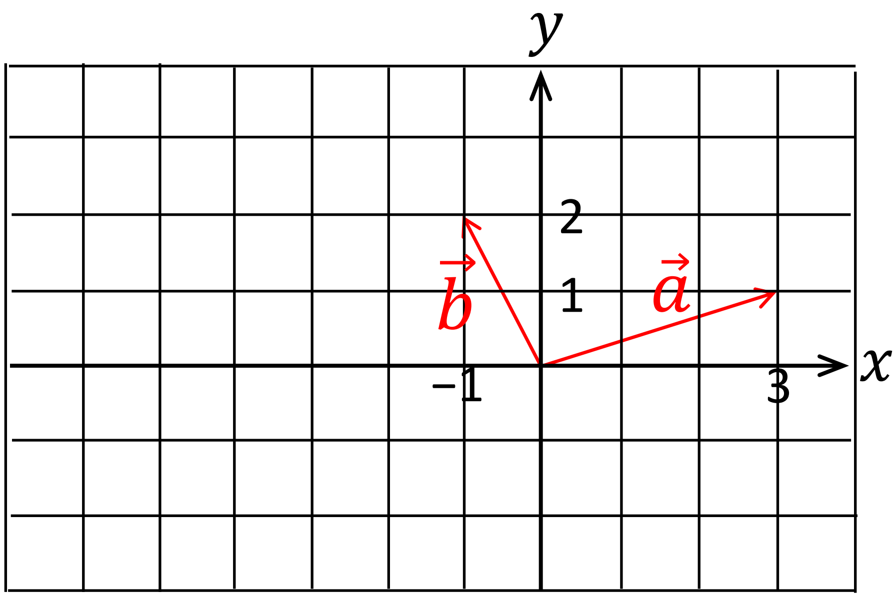
線形代数

$$f(x+y)=f(x)+f(y) \quad \cdots\cdots ①$$

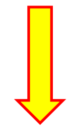
$$f(kx)=kf(x) \quad \cdots\cdots ②$$

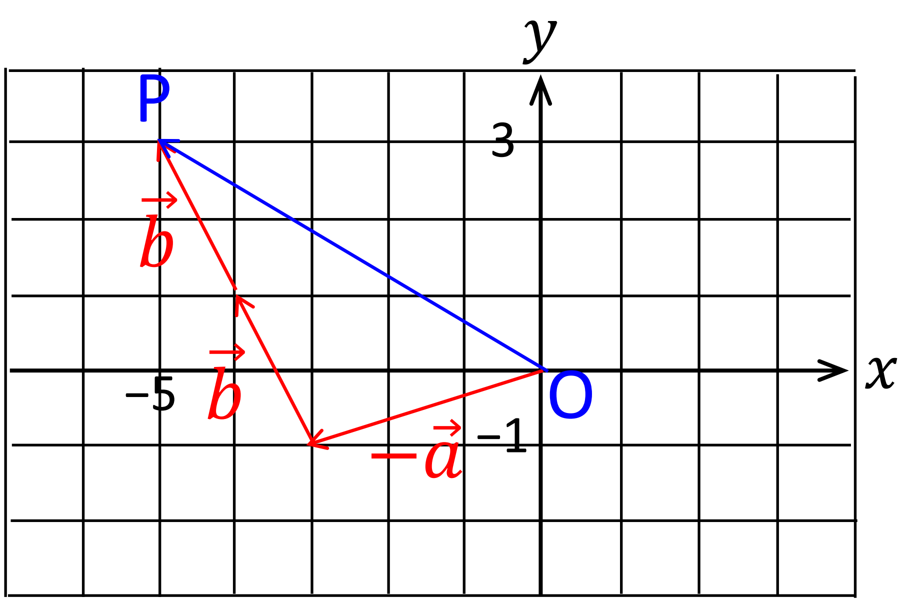






$\vec{a}=\begin{pmatrix}3\\1\end{pmatrix}$　　$\vec{b}=\begin{pmatrix}-1\\2\end{pmatrix}$



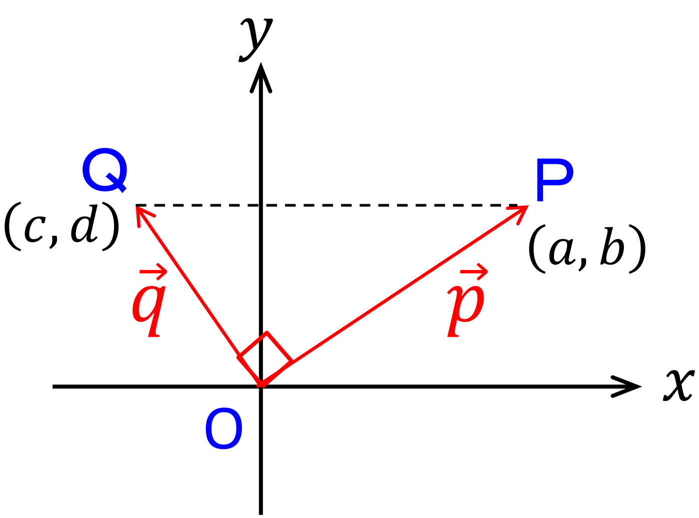


$$\overrightarrow{OP}=k\vec{a}+l\vec{b}$$

$k=-1, l=2$とすると

$$\overrightarrow{OP}=-\vec{a}+2\vec{b}$$
$$=-\begin{pmatrix}3\\1\end{pmatrix}+2\begin{pmatrix}-1\\2\end{pmatrix}$$
$$=\begin{pmatrix}-5\\3\end{pmatrix}$$

$\{\vec{a},\vec{b}\}$はR^2平面の基底という



$$\vec{p} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad \vec{q} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$OP^2 + OQ^2 = QP^2$$

$$\left|\overrightarrow{OP}\right|^2 + \left|\overrightarrow{OQ}\right|^2 - \left|\overrightarrow{QP}\right|^2 = 0$$

$$\vec{p} + \vec{q} - |\vec{p} - \vec{q}|$$

$$= \left|\begin{pmatrix} a \\ b \end{pmatrix}\right|^2 + \left|\begin{pmatrix} c \\ d \end{pmatrix}\right|^2 - \left|\begin{pmatrix} a-c \\ b-d \end{pmatrix}\right|^2$$

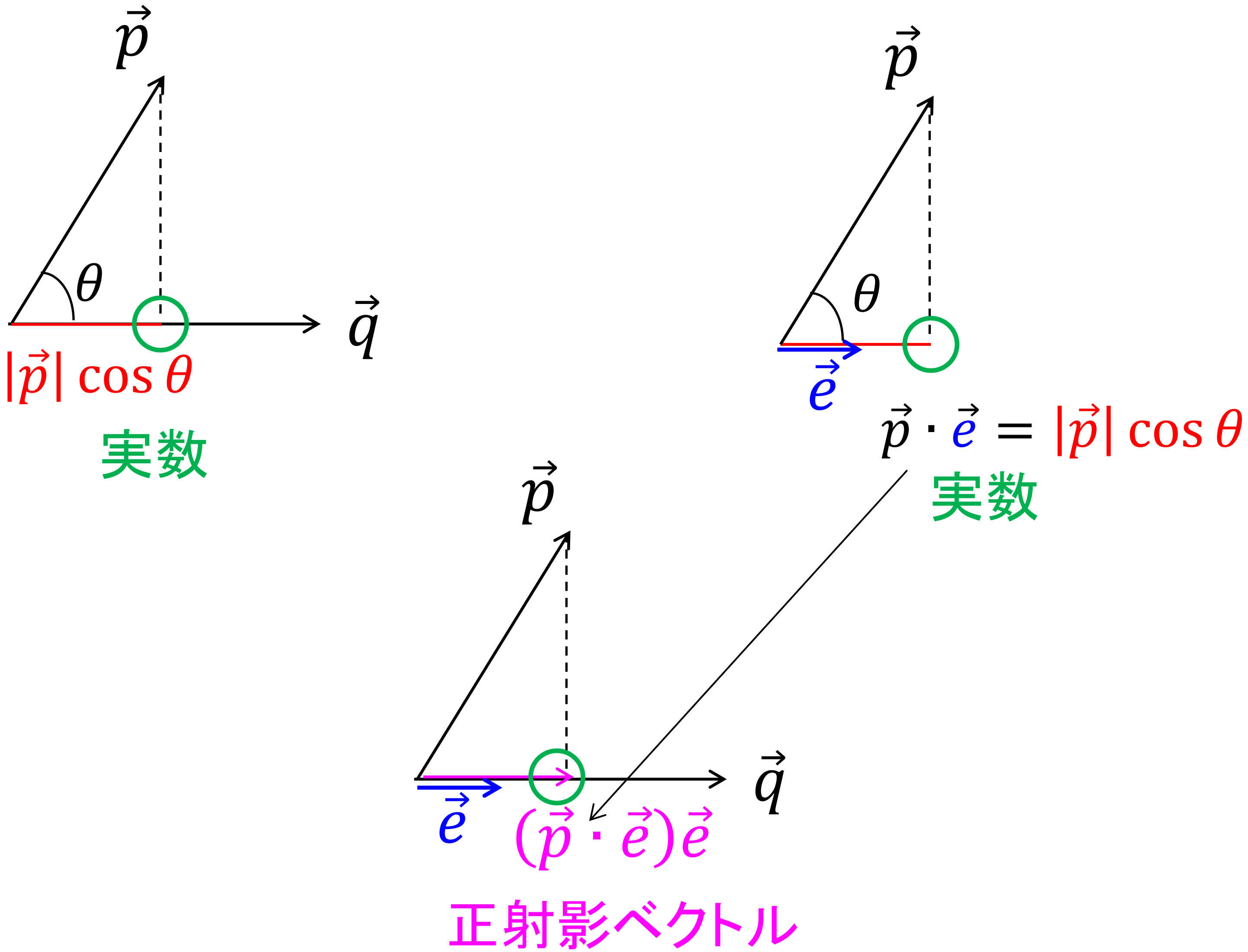
$$= (a^2 + b^2) + (c^2 + d^2) - \{(a-c)^2 + (b-d)^2\}$$

$$= 2ac+2bd = 0$$

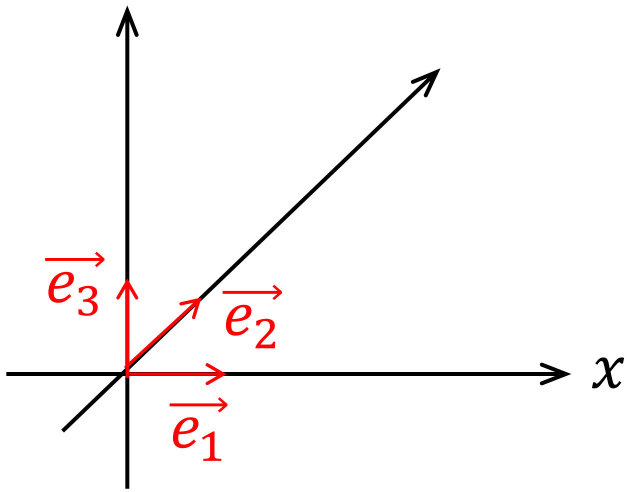
$$ac+bd = \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = \vec{p} \cdot \vec{q} = 0$$

内積　　　直交

$$\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\theta \qquad \vec{p}\cdot\vec{e} = |\vec{p}||\vec{e}|\cos\theta = |\vec{p}|\cos\theta$$



正規直交基底



正規

$\overrightarrow{e_1}\cdot\overrightarrow{e_1}=1$　$\overrightarrow{e_2}\cdot\overrightarrow{e_2}=1$　$\overrightarrow{e_3}\cdot\overrightarrow{e_3}=1$

直交

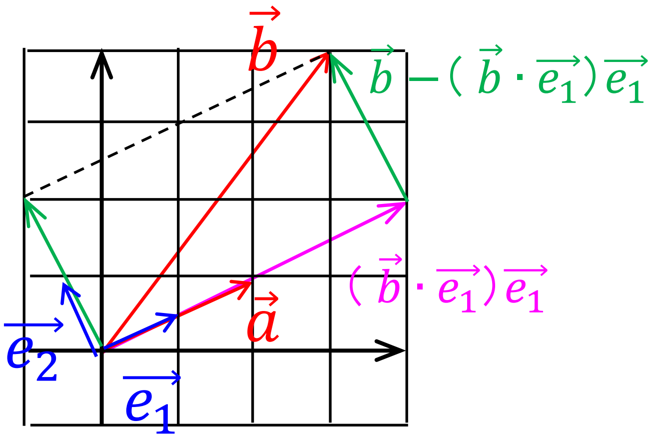
$\overrightarrow{e_1}\cdot\overrightarrow{e_2}=0$　$\overrightarrow{e_2}\cdot\overrightarrow{e_3}=0$　$\overrightarrow{e_3}\cdot\overrightarrow{e_1}=0$

$\vec{a}=\begin{pmatrix}2\\1\end{pmatrix}$　$\vec{b}=\begin{pmatrix}3\\4\end{pmatrix}$　から正規直交基底を作る

⇩

解答は次ページ

$\vec{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$



①$\vec{a}$の方向の単位ベクトル$\overrightarrow{e_1}$を算出する

$$\overrightarrow{e_1} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{\sqrt{2^2+1^2}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

② $\vec{b}$の$\overrightarrow{e_1}$方向の正射影ベクトルを算出する

$(\vec{b}\cdot\overrightarrow{e_1})\overrightarrow{e_1}$

③ $\vec{b}$と正射影ベクトルの差ベクトル

$\vec{b}-(\vec{b}\cdot\overrightarrow{e_1})\overrightarrow{e_1}$ は$\overrightarrow{e_1}$と直交する

$$\text{内積}(\vec{b}-(\vec{b}\cdot\overrightarrow{e_1})\overrightarrow{e_1})\cdot\overrightarrow{e_1} = \vec{b}\cdot\overrightarrow{e_1} - (\vec{b}\cdot\overrightarrow{e_1})\underbrace{\overrightarrow{e_1}\cdot\overrightarrow{e_1}}_{=1}$$

$$= \vec{b}\cdot\overrightarrow{e_1} - \vec{b}\cdot\overrightarrow{e_1} = 0$$

$$\vec{b}-(\vec{b}\cdot\overrightarrow{e_1})\overrightarrow{e_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \left\{\begin{pmatrix} 4 \\ 3 \end{pmatrix}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right\}\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \frac{3\cdot2+4\cdot1}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

④$\vec{b}-(\vec{b}\cdot\overrightarrow{e_1})\overrightarrow{e_1}$の単位ベクトル$\overrightarrow{e_2}$を算出する

$$\overrightarrow{e_2} = \frac{1}{\sqrt{(-1)^2+2^2}}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$\overrightarrow{e_1}\cdot\overrightarrow{e_2} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = 0$ 直交

$\overrightarrow{e_1}\cdot\overrightarrow{e_1} = 1$ $\quad \overrightarrow{e_2}\cdot\overrightarrow{e_2} = 1$ 正規