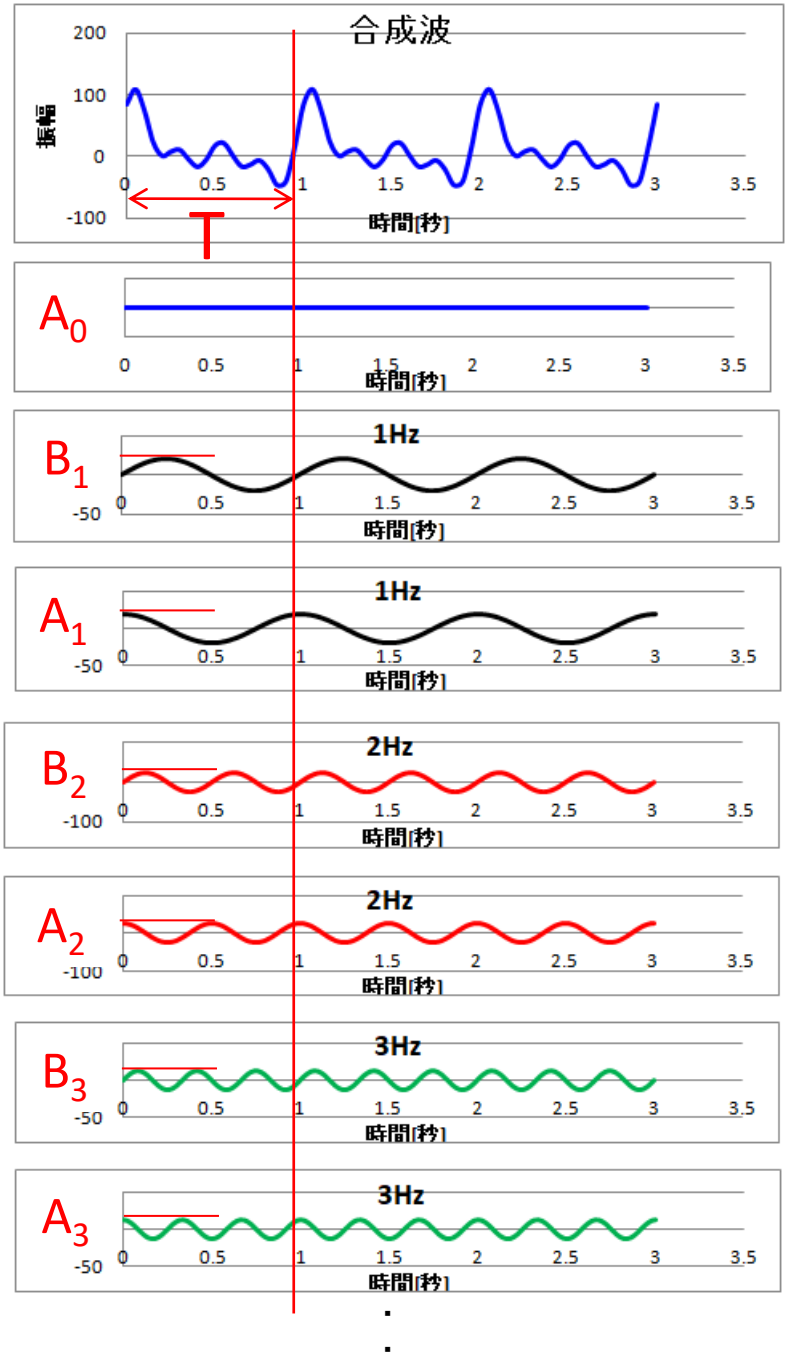


$f(t)$

A_0

$B_1 \sin \omega t$

$A_1 \cos \omega t$

$B_2 \sin \omega t$

$A_2 \cos \omega t$

$B_3 \sin \omega t$

$A_3 \cos \omega t$

$$A_0 = \frac{1}{T}\int_0^T f(t)dt$$

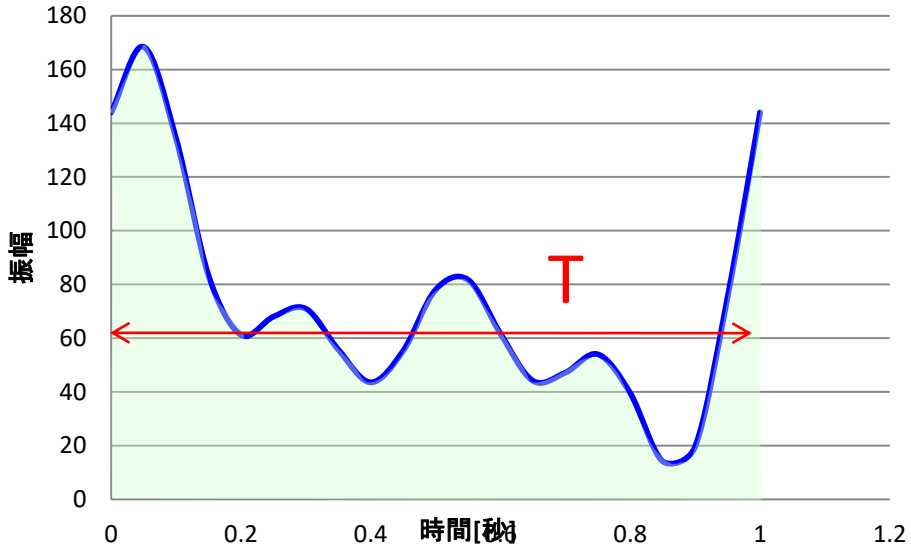
$$B_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

フーリエ展開

フーリエ級数

$$\begin{aligned} f(t) &= A_0 + \sum_{n=1}^{\infty} A_n \cos n\omega t + B_n \sin n\omega t \\ &= A_0 + A_1 \cos \omega t + B_1 \sin \omega t \\ &\quad + A_2 \cos 2\omega t + B_2 \sin 2\omega t \\ &\quad + A_3 \cos 3\omega t + B_3 \sin 3\omega t \\ &\quad + \cdots \end{aligned}$$



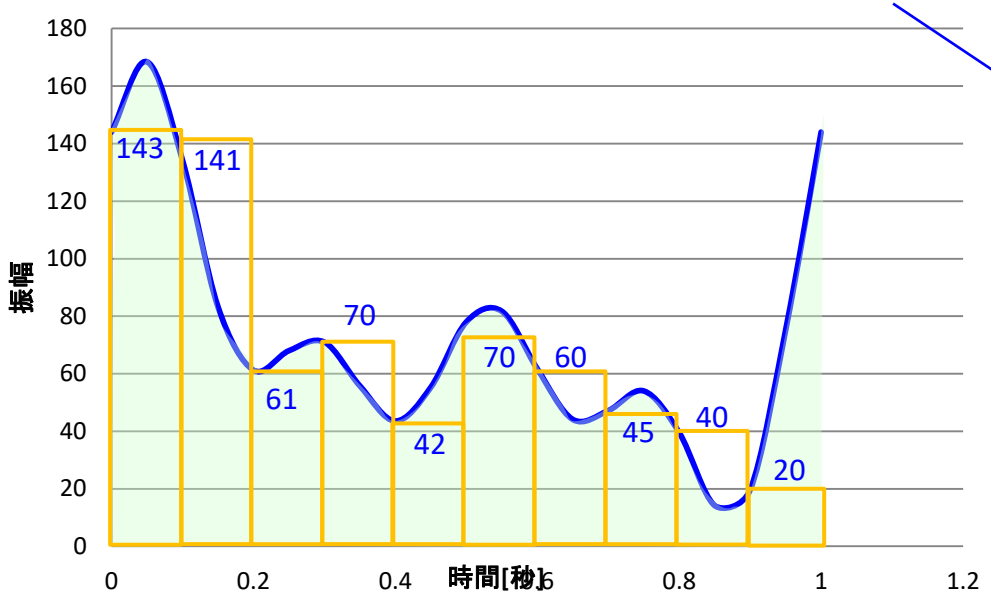
以下の式を使って周波数解析をしたいが、f(t)は実測のため関数がわからないことが多い。では、どうするか？

$$A_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

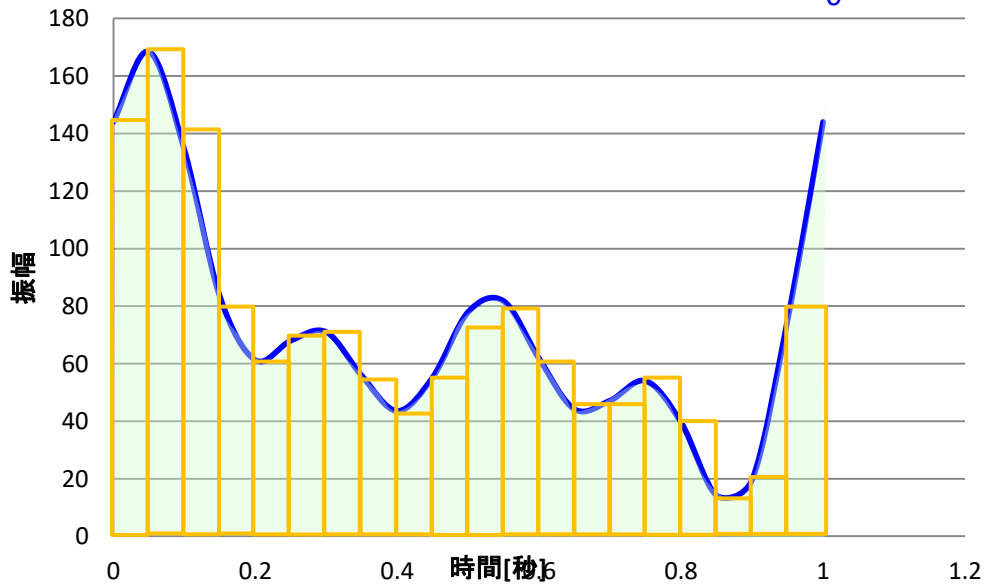
$$A_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

緑色の面積を求めるために、10分割して短冊の面積で代用
その面積を横軸の時間で割って平均の振幅A_0を算出



振幅	振幅×0.1
143	14.3
141	14.1
61	6.1
70	7
42	4.2
70	7
60	6
45	4.5
40	4
20	2
合計	69.2
A_0	69.2

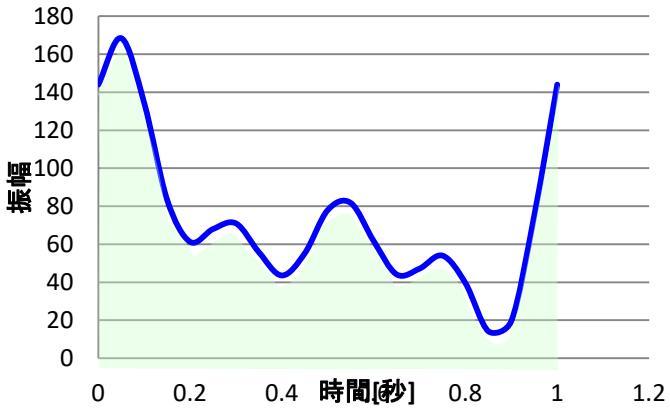
20分割して短冊の面積を算出 → 振幅A_0を算出



振幅	振幅×0.05
144	7.2
168	8.4
142	7.1
80	4
61	3.05
70	3.5
71	3.55
55	2.75
43	2.15
55	2.75
72	3.6
80	4
60	3
46	2.3
46	2.3
55	2.75
40	2
12	0.6
20	1
80	4
合計	70
A_0	70

	sin波				cos波					
周波数f	1	2	3	4	1	2	3	4		合成波
振幅A	20	25	13	16	20	25	13	16	A_0	
0	0	0	0	0	20	25	13	16	70	144
0.05	6.1803399	14.69463	10.51722093	15.2169	19.02113	20.22542	7.641208	4.944272	70	168.4411
0.1	11.755705	23.77641	12.36373471	9.404564	16.18034	7.725425	−4.01722	−12.9443	70	134.2447
0.15	16.18034	23.77641	4.017220927	−9.40456	11.75571	−7.72542	−12.3637	−12.9443	70	83.29168
0.2	19.02113	14.69463	−7.64120828	−15.2169	6.18034	−20.2254	−10.5172	4.944272	70	61.23962
0.25	20	3.06E−15	−13	−3.9E−15	1.23E−15	−25	−2.4E−15	16	70	68
0.3	19.02113	−14.6946	−7.64120828	15.2169	−6.18034	−20.2254	10.51722	4.944272	70	70.95792
0.35	16.18034	−23.7764	4.017220927	9.404564	−11.7557	−7.72542	12.36373	−12.9443	70	55.76404
0.4	11.755705	−23.7764	12.36373471	−9.40456	−16.1803	7.725425	4.017221	−12.9443	70	43.5565
0.45	6.1803399	−14.6946	10.51722093	−15.2169	−19.0211	20.22542	−7.64121	4.944272	70	55.29338
0.5	2.45E−15	−6.1E−15	4.77808E−15	−7.8E−15	−20	25	−13	16	70	78
0.55	−6.18034	14.69463	−10.5172209	15.2169	−19.0211	20.22542	−7.64121	4.944272	70	81.72133
0.6	−11.75571	23.77641	−12.3637347	9.404564	−16.1803	7.725425	4.017221	−12.9443	70	61.67957
0.65	−16.18034	23.77641	−4.01722093	−9.40456	−11.7557	−7.72542	12.36373	−12.9443	70	44.11262
0.7	−19.02113	14.69463	7.64120828	−15.2169	−6.18034	−20.2254	10.51722	4.944272	70	47.15353
0.75	−20	9.19E−15	13	−1.2E−14	−3.7E−15	−25	7.17E−15	16	70	54
0.8	−19.02113	−14.6946	7.64120828	15.2169	6.18034	−20.2254	−10.5172	4.944272	70	39.52432
0.85	−16.18034	−23.7764	−4.01722093	9.404564	11.75571	−7.72542	−12.3637	−12.9443	70	14.15286
0.9	−11.75571	−23.7764	−12.3637347	−9.40456	16.18034	7.725425	−4.01722	−12.9443	70	19.64386
0.95	−6.18034	−14.6946	−10.5172209	−15.2169	19.02113	20.22542	7.641208	4.944272	70	75.22294
1	−4.9E−15	−1.2E−14	−9.5562E−15	−1.6E−14	20	25	13	16	70	144

振幅	振幅×0.05
144	7.2
168	8.4
142	7.1
80	4
61	3.05
70	3.5
71	3.55
55	2.75
43	2.15
55	2.75
72	3.6
80	4
60	3
46	2.3
46	2.3
55	2.75
40	2
12	0.6
20	1
80	4
合計	70
A_0	70



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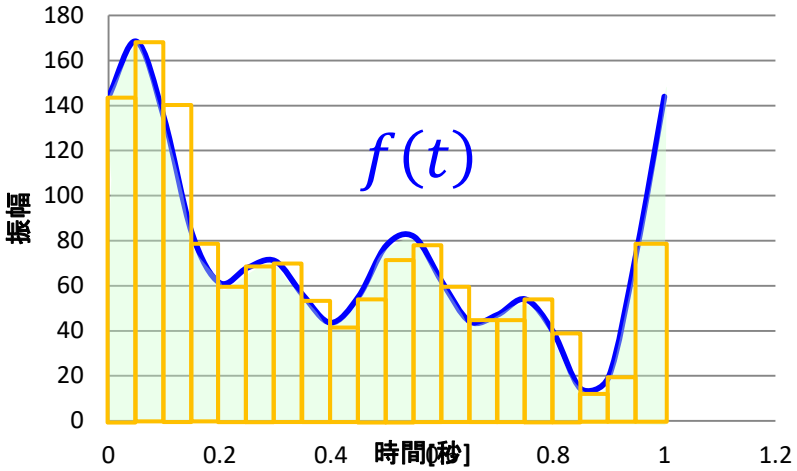
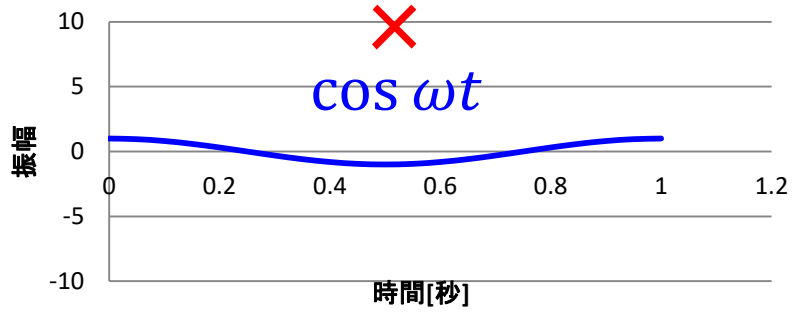
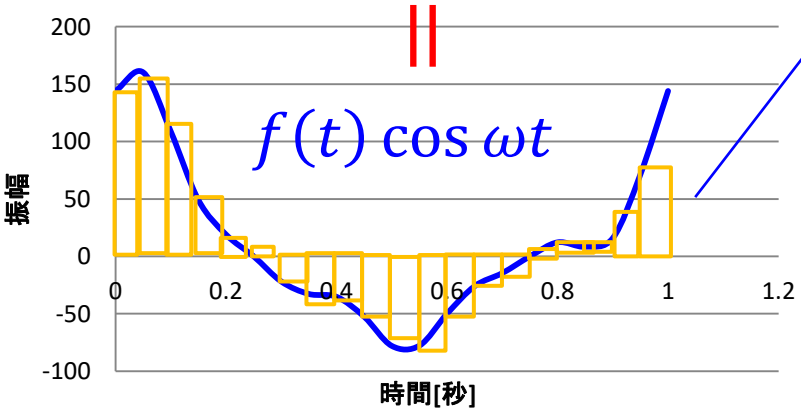






7.2
8.0
5.4
2.4
0.9
0.0
−1.1
−1.6
−1.8
−2.6
−3.9
−3.9
−2.5
−1.3
−0.7
0.0
0.6
0.4
0.8
3.6

合計	10
A_1	20

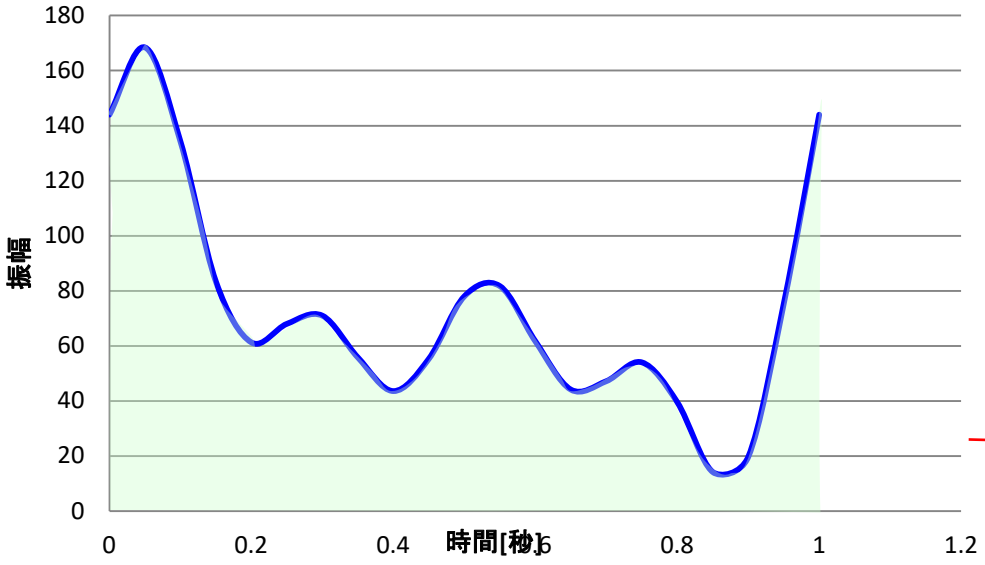
$$A_1 = \frac{2}{T}\int_0^T f(t)\cos \omega t \, dt$$

	sin波				cos波					
周波数f	1	2	3	4	1	2	3	4		合成波
振幅A	20	25	13	16	20	25	13	16	A_0	
0	0	0	0	0	20	25	13	16	70	144
0.05	6.1803399	14.69463	10.51722093	15.2169	19.02113	20.22542	7.641208	4.944272	70	168.4411
0.1	11.755705	23.77641	12.36373471	9.404564	16.18034	7.725425	−4.01722	−12.9443	70	134.2447
0.15	16.18034	23.77641	4.017220927	−9.40456	11.75571	−7.72542	−12.3637	−12.9443	70	83.29168
0.2	19.02113	14.69463	−7.64120828	−15.2169	6.18034	−20.2254	−10.5172	4.944272	70	61.23962
0.25	20	3.06E−15	−13	−3.9E−15	1.23E−15	−25	−2.4E−15	16	70	68
0.3	19.02113	−14.6946	−7.64120828	15.2169	−6.18034	−20.2254	10.51722	4.944272	70	70.95792
0.35	16.18034	−23.7764	4.017220927	9.404564	−11.7557	−7.72542	12.36373	−12.9443	70	55.76404
0.4	11.755705	−23.7764	12.36373471	−9.40456	−16.1803	7.725425	4.017221	−12.9443	70	43.5565
0.45	6.1803399	−14.6946	10.51722093	−15.2169	−19.0211	20.22542	−7.64121	4.944272	70	55.29338
0.5	2.45E−15	−6.1E−15	4.77808E−15	−7.8E−15	−20	25	−13	16	70	78
0.55	−6.18034	14.69463	−10.5172209	15.2169	−19.0211	20.22542	−7.64121	4.944272	70	81.72133
0.6	−11.75571	23.77641	−12.3637347	9.404564	−16.1803	7.725425	4.017221	−12.9443	70	61.67957
0.65	−16.18034	23.77641	−4.01722093	−9.40456	−11.7557	−7.72542	12.36373	−12.9443	70	44.11262
0.7	−19.02113	14.69463	7.64120828	−15.2169	−6.18034	−20.2254	10.51722	4.944272	70	47.15353
0.75	−20	9.19E−15	13	−1.2E−14	−3.7E−15	−25	7.17E−15	16	70	54
0.8	−19.02113	−14.6946	7.64120828	15.2169	6.18034	−20.2254	−10.5172	4.944272	70	39.52432
0.85	−16.18034	−23.7764	−4.01722093	9.404564	11.75571	−7.72542	−12.3637	−12.9443	70	14.15286
0.9	−11.75571	−23.7764	−12.3637347	−9.40456	16.18034	7.725425	−4.01722	−12.9443	70	19.64386
0.95	−6.18034	−14.6946	−10.5172209	−15.2169	19.02113	20.22542	7.641208	4.944272	70	75.22294
1	−4.9E−15	−1.2E−14	−9.5562E−15	−1.6E−14	20	25	13	16	70	144

一致

7.2
8.0
5.4
2.4
0.9
0.0
−1.1
−1.6
−1.8
−2.6
−3.9
−3.9
−2.5
−1.3
−0.7
0.0
0.6
0.4
0.8
3.6

合計	10
A_1	20

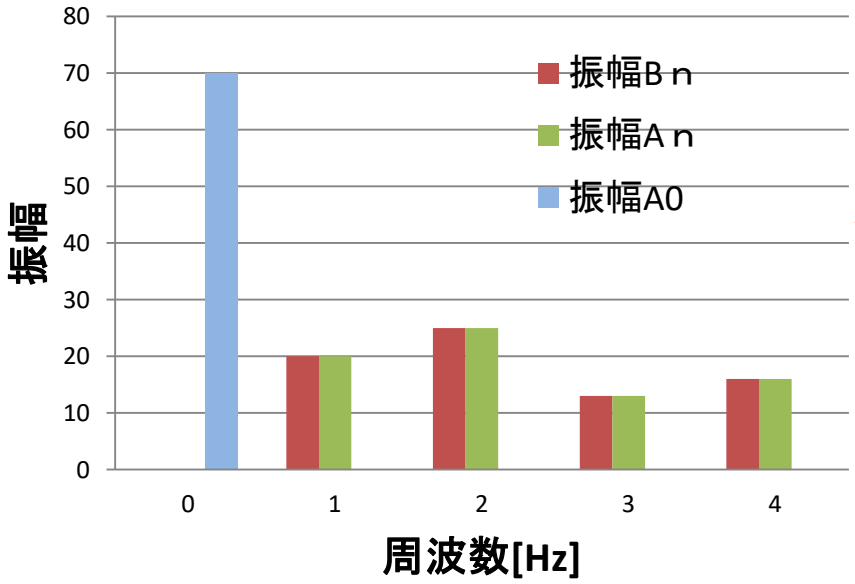


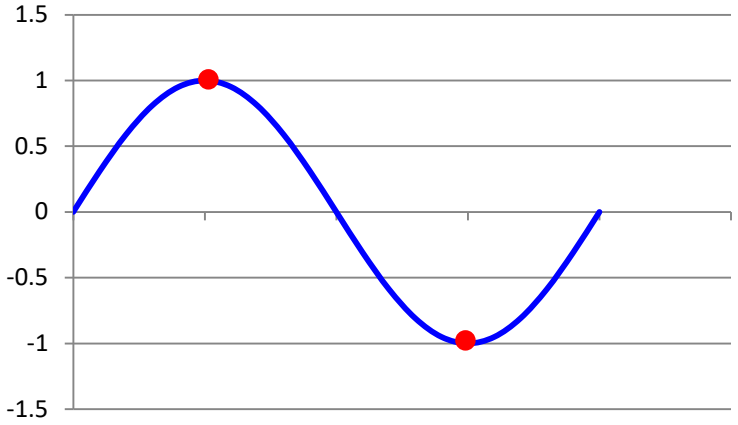
$$A_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

フーリエ展開





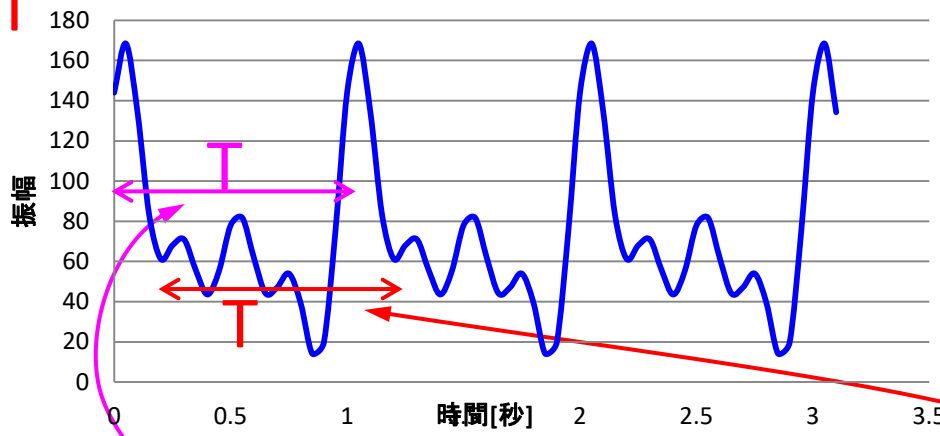
２点のデータがあれば、上下に１回振動
10点のデータがあれば、上下に5回振動の波まで求めることが可能

連続のデータ全てを使わなくても、適切な短冊の幅を決めて計算すれば、処理スピードは上がる。 このように飛び飛びの値を使うフーリエ展開のことを

離散フーリエ展開

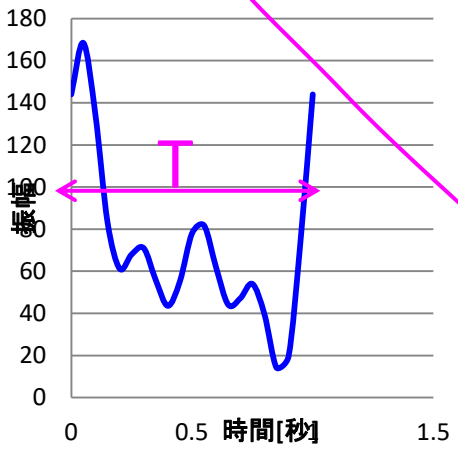
と呼びます

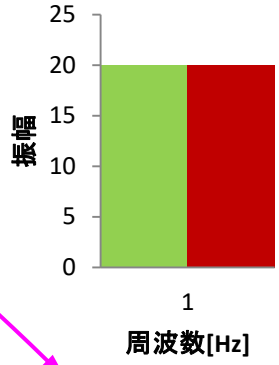
T= T



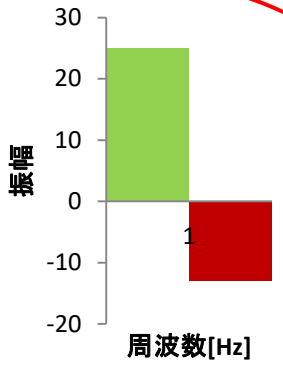
同じ周期TあるいはTであっても、サンプリング期間が異なると、波の形が異なりフーリエ展開で算出のA_nあるいはB_nは異なる。
そこで周波数特性は、

$P = \sqrt{(A_n{}^2 + B_n{}^2)}$で算出する

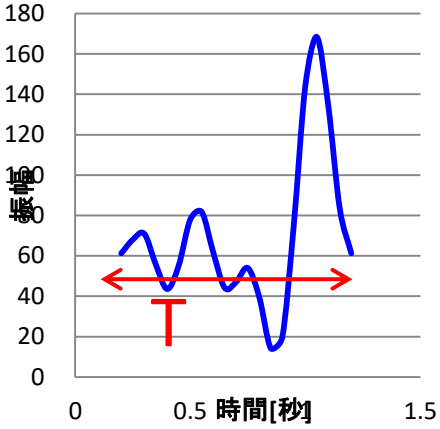


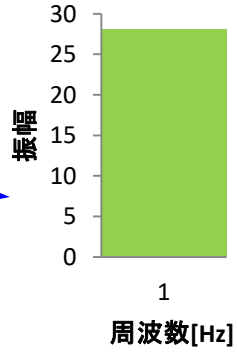


$A_1 = 20$
$B_1 = 20$



$A_1 = 25$
$B_1 = -13$





$P = \sqrt{(A_1{}^2 + B_1{}^2)}$
$= \sqrt{(20^2 + 20^2)}$
$=28$

$P = \sqrt{(A_1{}^2 + B_1{}^2)}$
$= \sqrt{(25^2 + (-13)^2)}$
$=28$