相関係数

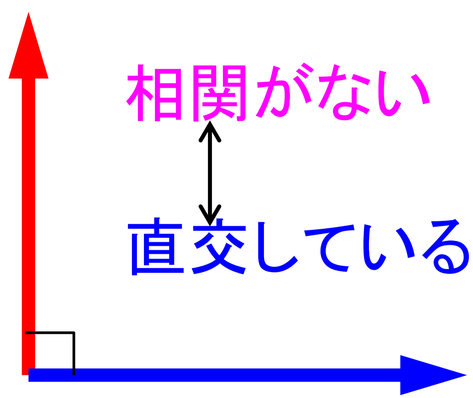
ー1≦相関係数r≦ 1

X' Y'

$$\begin{pmatrix} x_1-\bar{x} \\ x_2-\bar{x} \\ x_3-\bar{x} \\ x_4-\bar{x} \\ x_5-\bar{x} \\ x_6-\bar{x} \\ x_7-\bar{x} \\ x_8-\bar{x} \\ \vdots \\ x_{N\text{-}3}-\bar{x} \\ x_{N\text{-}3}-\bar{x} \\ x_{N\text{-}2}-\bar{x} \\ x_{N\text{-}1}-\bar{x} \\ x_N-\bar{x} \end{pmatrix} \quad \begin{pmatrix} y_1-\bar{y} \\ y_2-\bar{y} \\ y_3-\bar{y} \\ y_4-\bar{y} \\ y_5-\bar{y} \\ y_6-\bar{y} \\ y_7-\bar{y} \\ y_8-\bar{y} \\ \vdots \\ y_{N\text{-}3}-\bar{y} \\ y_{N\text{-}3}-\bar{y} \\ y_{N\text{-}2}-\bar{y} \\ y_{N\text{-}1}-\bar{y} \\ y_N-\bar{y} \end{pmatrix}$$

θ=0° cosθ=1 r=1

θ=90° cosθ=0 r=0



相関がない

↕

直交している

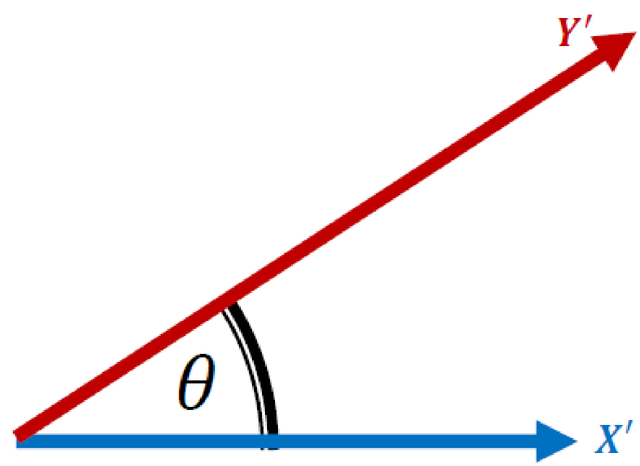
相関係数 r ベクトルX'、Y'の**内積**

$$= \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2}\sqrt{\sum(y_i-\bar{y})^2}}$$

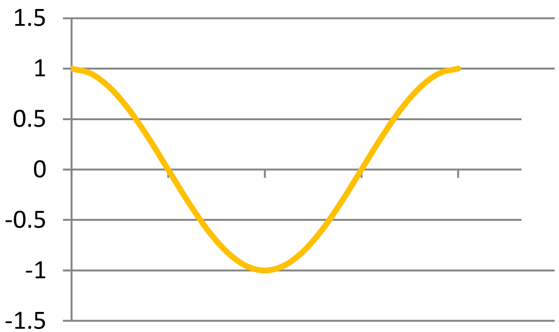
ベクトルX'、Y'の**ノルム**
（スカラー、長さ・大きさ）

$$= \frac{X'\cdot Y'}{\|X'\|\|Y'\|}$$

$$= \cos\theta$$

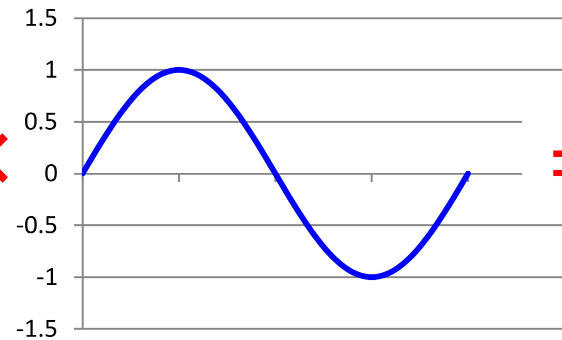


$\cos \omega t$



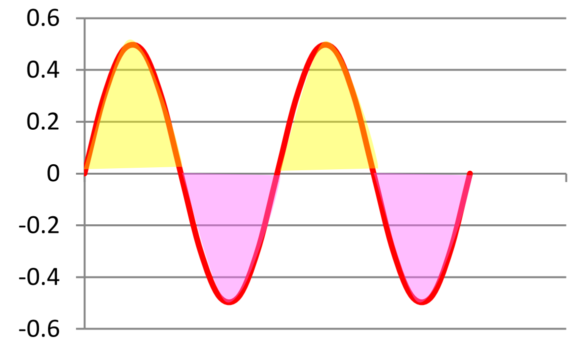
$\times$

$\sin \omega t$



$=$

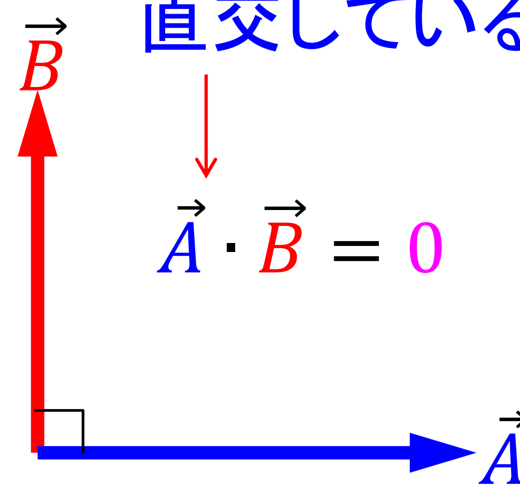
$\cos \omega t \cdot \sin \omega t$



$$\int_0^T \cos \omega t \cdot \sin \omega t \, dt = 0$$

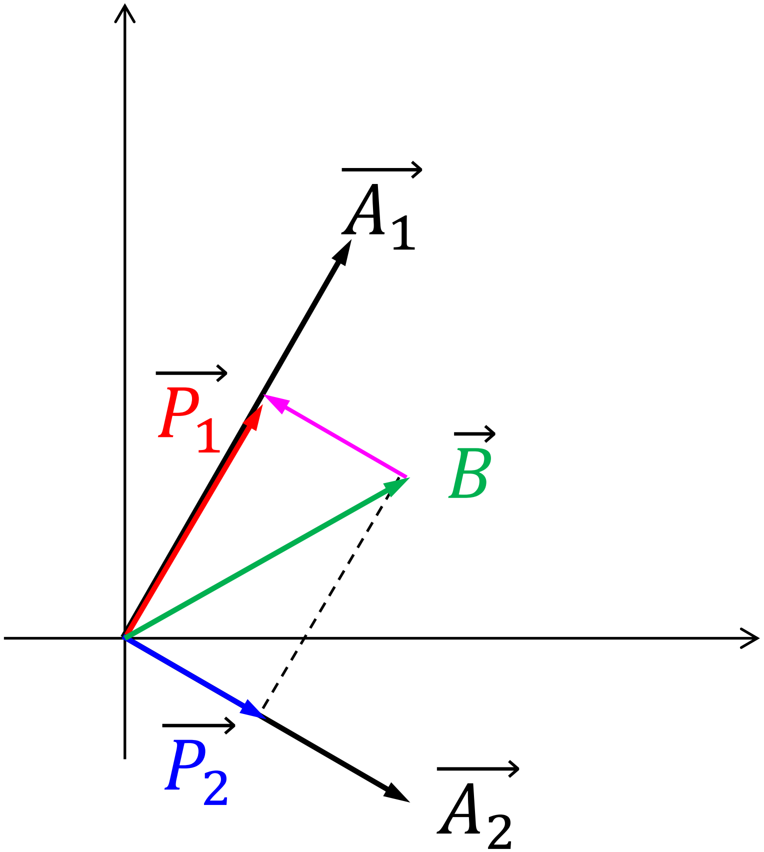
直交している

$$\vec{A} \cdot \vec{B} = 0$$



ベクトルの内積は積分で表せる

$$\vec{A} \cdot \vec{B} = \int_0^T \cos \omega t \cdot \sin \omega t \, dt$$



$$\overrightarrow{A_1} \cdot (\overrightarrow{P_1} - \overrightarrow{B}) = 0$$

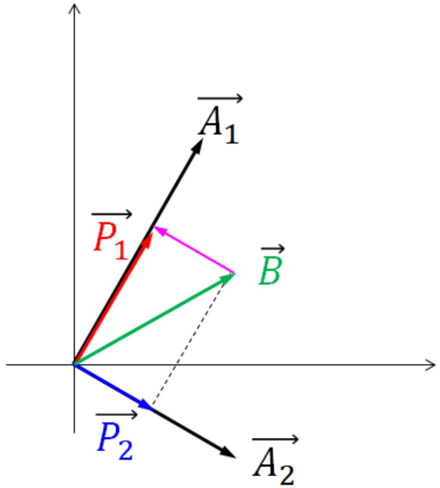
$\overrightarrow{P_1} = x\overrightarrow{A_1}$を代入

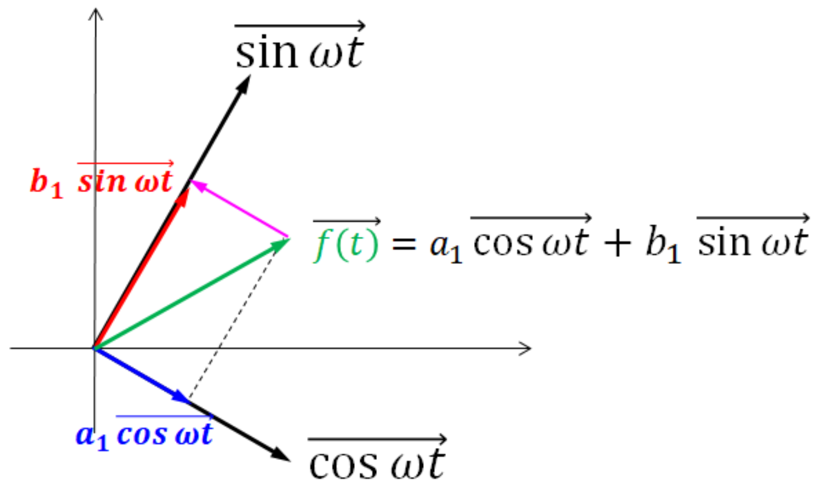
$$\begin{aligned} &\overrightarrow{A_1} \cdot (\overrightarrow{P_1} - \overrightarrow{B}) \\ &= \overrightarrow{A_1} \cdot (x\overrightarrow{A_1} - \overrightarrow{B}) \\ &= x\overrightarrow{A_1} \cdot \overrightarrow{A_1} - \overrightarrow{A_1} \cdot \overrightarrow{B} \\ &= 0 \end{aligned}$$

$$x = \frac{\overrightarrow{A_1} \cdot \overrightarrow{B}}{\overrightarrow{A_1} \cdot \overrightarrow{A_1}}$$

$\overrightarrow{B}$の$\overrightarrow{A_2}$軸への射影が$\overrightarrow{P_2}$
$\overrightarrow{A_1}$軸への射影が$\overrightarrow{P_1}$

$$\overrightarrow{P_1} = \frac{\overrightarrow{A_1} \cdot \overrightarrow{B}}{\overrightarrow{A_1} \cdot \overrightarrow{A_1}} \overrightarrow{A_1}$$





$$\overrightarrow{P_1} = \frac{\overrightarrow{A_1}\cdot\vec{B}}{\overrightarrow{A_1}\cdot\overrightarrow{A_1}}\overrightarrow{A_1}$$

$$\overrightarrow{A_1}\cdot\vec{B} = \int_0^T \sin\omega t\cdot f(t)dt$$

$$\overrightarrow{A_1}\cdot\overrightarrow{A_1} = \int_0^T \sin\omega t\cdot\sin\omega t\,dt$$

$$b_1\ \overrightarrow{\sin\omega t} = \frac{\int_0^T \sin\omega t\cdot f(t)dt}{\int_0^T \sin\omega t\cdot\sin\omega t dt}\overrightarrow{\sin\omega t}$$

$$b_1 = \frac{\int_0^T \sin\omega t\cdot f(t)dt}{\int_0^T \sin\omega t\cdot\sin\omega t\,dt}$$

$$= \frac{2}{T}\int_0^T \sin\omega t\cdot f(t)\,dt$$

$$\overrightarrow{P_1} = \frac{\overrightarrow{A_2}\cdot\vec{B}}{\overrightarrow{A_2}\cdot\overrightarrow{A_2}}\overrightarrow{A_2}$$

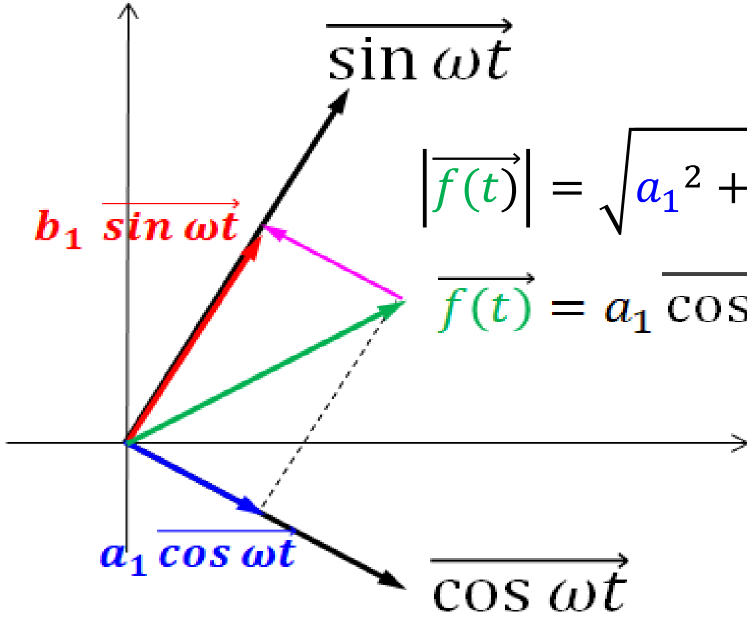
$$\overrightarrow{A_2}\cdot\vec{B} = \int_0^T \cos\omega t\cdot f(t)dt$$

$$\overrightarrow{A_2}\cdot\overrightarrow{A_2} = \int_0^T \cos\omega t\cdot\cos\omega t\ \ dt$$

$$a_1\ \overrightarrow{\cos\omega t} = \frac{\int_0^T \cos\omega t\cdot f(t)dt}{\int_0^T \cos\omega t\cdot\cos\omega t\ \ dt}\overrightarrow{\cos\omega t}$$

$$a_1 = \frac{\int_0^T \cos\omega t\cdot f(t)dt}{\int_0^T \cos\omega t\cdot\cos\omega t\ \ dt}$$

$$= \frac{2}{T}\int_0^T \cos\omega t\cdot f(t)\,dt$$

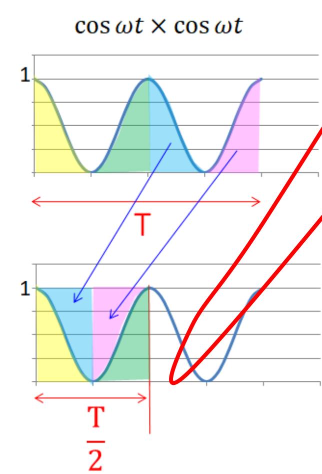


$$\left|\overrightarrow{f(t)}\right| = \sqrt{{a_1}^2 + {b_1}^2}$$

$$\overrightarrow{f(t)} = a_1\ \overrightarrow{\cos \omega t} + b_1\ \overrightarrow{\sin \omega t}$$

$$\boldsymbol{b_1} = \frac{\int_0^T \sin \omega t \cdot f(t)dt}{\int_0^T \sin \omega t \cdot \sin \omega t\, dt} = \frac{2}{T}\int_0^T \sin \boldsymbol{\omega t} \cdot \boldsymbol{f(t)\, dt}$$

$$\boldsymbol{a_1} = \frac{\int_0^T \cos \omega t \cdot f(t)dt}{\boxed{\int_0^T \cos \omega t \cdot \cos \omega t\ dt}} = \boxed{\frac{2}{T}}\int_0^T \cos \boldsymbol{\omega t} \cdot \boldsymbol{f(t)\, dt}$$



関数f(t)のcosωt軸への射影の成分がa_1とsinωt軸への射影の成分がb_1
ベクトルf(t)の大きさが

$$\sqrt{{a_1}^2 + {b_1}^2}$$