$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$ フーリエ級数

$$a_0 = \frac{1}{T}\int_0^T f(t)\ dt$$
$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\ dt$$
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\ dt$$

係数の算出式

オイラーの公式の変形

$$\cos n\omega t = \frac{e^{in\omega t} + e^{-in\omega t}}{2} \qquad \sin n\omega t = \frac{e^{in\omega t} - e^{-in\omega t}}{2i}$$

書換え

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(\frac{a_n}{2}\left(e^{in\omega t} + e^{-in\omega t}\right) + \frac{b_n}{2i}\left(e^{in\omega t} - e^{-in\omega t}\right)\right)$$
$$a_0 = \frac{1}{T}\int_0^T f(t)\ dt$$
$$a_n = \frac{2}{T}\int_0^T f(t)\ \frac{1}{2}\left(e^{in\omega t} + e^{-in\omega t}\right)dt$$
$$b_n = \frac{2}{T}\int_0^T f(t)\frac{1}{2i}\left(e^{in\omega t} - e^{-in\omega t}\right)dt$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(\underline{\frac{a_n}{2}\left(e^{in\omega t} + e^{-in\omega t}\right)} + \underline{\frac{b_n}{2i}\left(e^{in\omega t} - e^{-in\omega t}\right)} \right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\ dt$$

$$a_n = \frac{1}{T}\int_0^T f(t)\ \left(e^{in\omega t} + e^{-in\omega t}\right)dt$$

$$ib_n = \frac{1}{T}\int_0^T f(t)\ \left(e^{in\omega t} - e^{-in\omega t}\right)dt$$

$\frac{1}{2}(a_n - ib_n) = A_n$ $\quad$ $\frac{1}{2}(a_n + ib_n) = B_n$ とおく

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(\underline{A_n\, e^{-in\omega t}} + \underline{B_n\, e^{in\omega t}} \right)$$

$$A_n = \frac{1}{T}\int_0^T f(t) e^{-in\omega t}\, dt$$

$$B_n = \frac{1}{T}\int_0^T f(t)\ e^{in\omega t} dt$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(A_n\, e^{-in\omega t} + B_n\, e^{in\omega t} \right)$$

$$A_n = \frac{1}{T} \int_0^T f(t) e^{-in\omega t}\, dt$$

$$B_n = \frac{1}{T} \int_0^T f(t)\; e^{in\omega t} dt$$

$$A_0 = \frac{1}{T} \int_0^T f(t) e^{-i0\omega t}\, dt = \frac{1}{T} \int_0^T f(t)\; dt = a_0$$

$$B_{(-n)} = \frac{1}{T} \int_0^T f(t)\; e^{i(-n)\omega t} dt = \frac{1}{T} \int_0^T f(t)\; e^{-in\omega t} dt = A_n$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-in\omega t} \qquad C_n = \frac{1}{T} \int_0^T f(t) e^{-in\omega t}\, dt$$

$$C_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega t}\,dt$$

$\omega = 2\pi f$ とおく

$$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-i2\pi ft}\,dt$$

周期Ｔを持たない
関数にも拡張

Tに∞を代入すると、$\frac{T}{2} = \frac{\infty}{2} = \infty$ 、 $-\frac{T}{2} = \frac{-\infty}{2} = -\infty$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-i2\pi ft}$$

$$= \sum_{n=-\infty}^{\infty} \left\{\frac{1}{T}\int_{-\infty}^{\infty} f(t)e^{-i2\pi ft}\,dt\right\} e^{-i2\pi ft}$$

$$f(t) = \lim_{T\to\infty} \sum_{n=-\infty}^{\infty} \left\{\frac{1}{T}\int_{-\infty}^{\infty} f(t)e^{-i2\pi ft}\,dt\right\} e^{-i2\pi ft}$$

周期$T \to \infty$　　$\frac{1}{T} = f$　なので　$f \to \Delta f$

$$f(t) = \lim_{T \to \infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{1}{T} \int_{-\infty}^{\infty} f(t) e^{-i2\pi f t}\, dt \right\} e^{-i2\pi f t}$$

$$= \lim_{T \to \infty} \sum_{n=-\infty}^{\infty} \{ \Delta f \int_{-\infty}^{\infty} f(t) e^{-i2\pi f t}\, dt \} e^{-i2\pi f t}$$

$$= \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} f(t) e^{-i2\pi f_n t}\, dt \} e^{-i2\pi f t}\, df$$

周期Ｔを持たない
関数にも拡張

$$G(f) = \int_{-\infty}^{\infty} f(t) e^{-i2\pi f t}\, dt$$　フーリエ変換

$$f(t) = \int_{-\infty}^{\infty} G(f) e^{-i2\pi f t}\, df$$　フーリエ逆変換

$$C_n = \frac{1}{T} \int_{0}^{\mathrm{T}} f(t) e^{-i2\pi f_n t}\, dt$$　フーリエ展開