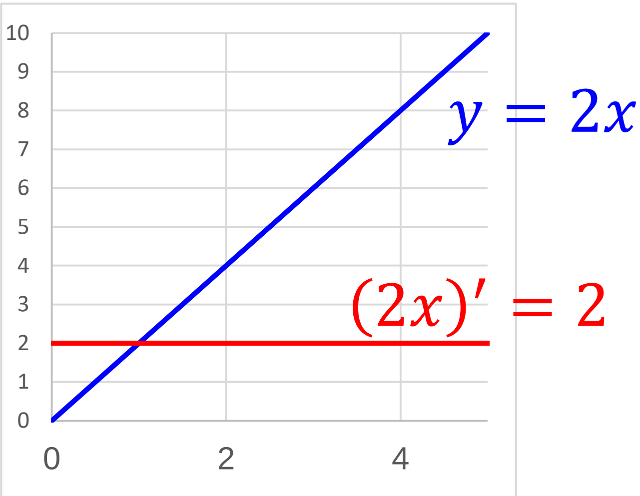

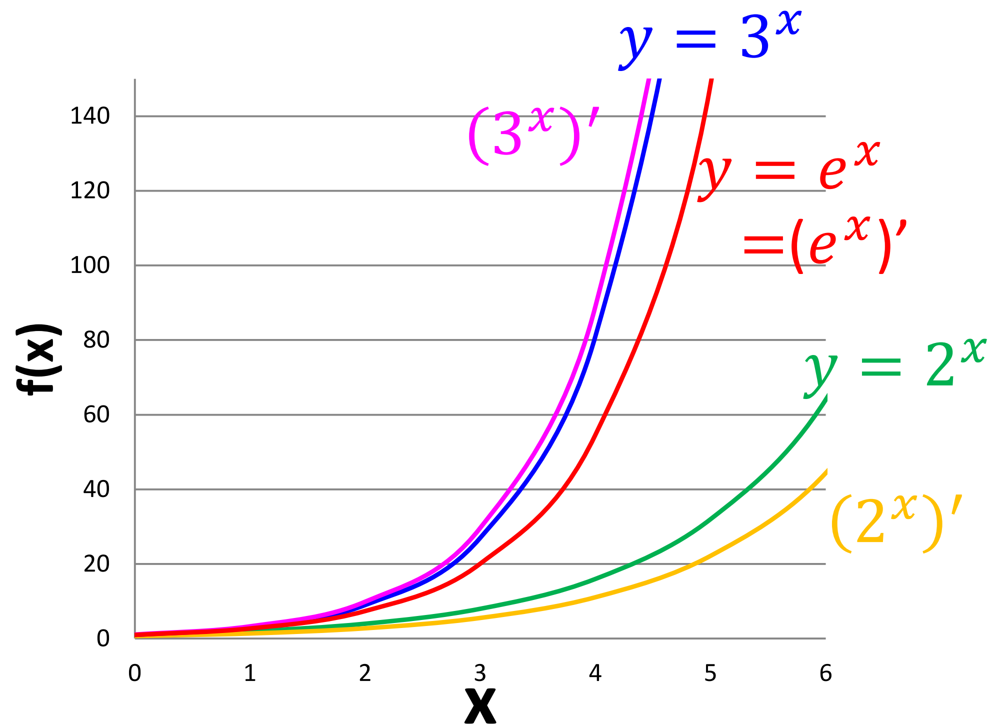

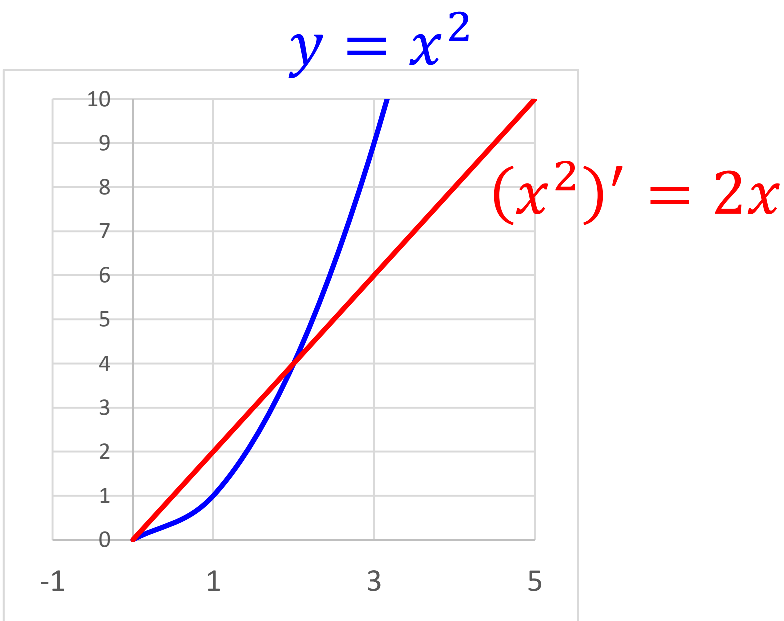

$$(a^x)' = (\ln a)a^x$$

$a = e$を代入すると

$$(e^x)' = (\ln e)e^x$$
$$= e^x$$

$$e^x \xleftrightarrow[\text{積分}]{\text{微分}} e^x$$

$$f(x) = e^{ax}$$
$$f'(x) = a \cdot e^{ax}$$
$$f''(x) = a^2 \cdot e^{ax}$$
$$f'''(x) = a^3 \cdot e^{ax}$$
$$\cdots$$
$$\cdots$$
$$\cdots$$
$$f^{(n)}(x) = a^n \cdot e^{ax}$$

$$f(x) = e^{ix}$$
$$f'(x) = i \cdot e^{ix}$$
$$f''(x) = -e^{ix}$$
$$f'''(x) = -ie^{ix}$$
$$f^{(4)}(x) = e^{ix}$$
$$\cdots$$
$$\cdots$$

マクローリン展開

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots\cdots$$

$$= a_0 + f'(0) + \frac{f'(0)}{2!} x^2 + \frac{f''(0)}{3!} x^3 + \frac{f'''(0)}{4!} x^4 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(0) x^n$$

係数a_0、a_1、a_2、a_3 $\cdots\cdots$を求めるには？

a_0の場合は、$x = 0$を代入して$f(0) = a_0$

a_1の場合は、

$$f'(x) = 0 + a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots\cdots$$

$x = 0$を代入して$f'(0) = a_1$

a_2の場合は、

$$f''(x) = 0 + 1 \cdot 2a_2 + 2 \cdot 3a_3 x + 3 \cdot 4a_4 x^2 + \cdots\cdots$$

$x = 0$を代入して$f''(0) = 1 \cdot 2a_2$

$$a_2 = \frac{f'(0)}{1 \cdot 2}$$

a_3の場合は、同様に

$$a_3 = \frac{f''(0)}{1 \cdot 2 \cdot 3}$$

$$a_n = \frac{f^{(n)}(0)}{n\,!}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(0) x^n$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^n(0) x^n$$

$$f(x) = \cos x$$
$$f'(x) = -\sin x$$
$$f''(x) = -\cos x$$
$$f'''(x) = \sin x$$
$$f^4(x) = \cos x$$

$$f(x) = \sin x$$
$$f'(x) = \cos x$$
$$f''(x) = -\sin x$$
$$f'''(x) = -\cos x$$
$$f^4(x) = \sin x$$

$$a_0 = \frac{f(0)}{0\ !} = \frac{1}{0\ !} \cdot 1 = 1$$
$$a_1 = \frac{f'(0)}{1\ !} = \frac{1}{1\ !} \cdot 0 = 0$$
$$a_2 = \frac{f''(0)}{2\ !} = \frac{1}{2\ !} \cdot -1 = -\frac{1}{2\ !}$$
$$a_3 = \frac{f'''(0)}{3\ !} = \frac{1}{3\ !} \cdot 0 = 0$$
$$a_4 = \frac{f^4(0)}{4\ !} = \frac{1}{4\ !} \cdot 1 = \frac{1}{4\ !}$$

$$a_0 = \frac{f(0)}{0\ !} = \frac{1}{0\ !} \cdot 0 = 0$$
$$a_1 = \frac{f'(0)}{1\ !} = \frac{1}{1\ !} \cdot 1 = \frac{1}{1\ !}$$
$$a_2 = \frac{f''(0)}{2\ !} = \frac{1}{2\ !} \cdot 0 = 0$$
$$a_3 = \frac{f'''(0)}{3\ !} = \frac{1}{3\ !} \cdot -1 = -\frac{1}{3\ !}$$
$$a_4 = \frac{f^4(0)}{4\ !} = \frac{1}{4\ !} \cdot 0 = 0$$

$f(x) = \cos x + i\sin x$
$f'(x) = -\sin x + i\cos x$
$f''(x) = -\cos x - i\sin x$
$f'''(x) = \sin x - i\cos x$
$f^4(x) = \cos x + i\sin x$

$f(x) = e^{ix}$
$f'(x) = i \cdot e^{ix}$
$f''(x) = -e^{ix}$
$f'''(x) = -ie^{ix}$
$f^4(x) = e^{ix}$

$f(0) = \cos 0 + i\sin 0 = 1$
$f'(0) = -\sin 0 + i\cos 0 = i$
$f''(0) = -\cos 0 - i\sin 0 = -1$
$f'''(0) = \sin 0 - i\cos 0 = -i$
$f^4(0) = \cos 0 + i\sin 0 = 1$

$f(0) = e^{i0} = 1$
$f'(0) = i \cdot e^{i0} = i$
$f''(0) = -e^{i0} = -1$
$f'''(0) = -ie^{i0} = -i$
$f^4(0) = e^{i0} = 1$

$$e^{ix} = \cos x + i\sin x$$

オイラーの公式

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$